Transfer function and state space representation of electric RLC circuit.

For electric RLC circuit shown above dynamic models will be designated. First dynamic model will be in form of transfer function. Second dynamic model will be in form of state space representation equations. Start conditions for this example are equal to zero \((ST' = 0)\). Circuit’s output voltage(output signal) is capacitor \(C\) voltage \(u_c(t)\).

1. Dynamic model of circuit in form transfer function \(H(s)\).

Circuit is treated like voltage divider. We calculate its resultant impedance in Laplace transformation form.

We assume start conditions of transformation as equal to zero \((ST' = 0)\).

\[
Z(s) = R + s \cdot L + \frac{1}{s \cdot C}
\]
Output voltage $U_2(s)$ will be the equal to

$$U_2(s) = \frac{1}{\frac{s \cdot C}{R + \frac{1}{s \cdot C} + s \cdot L}} \cdot U_1(s)$$

$$H(s) = \frac{U_2(s)}{U_1(s)} = \frac{1}{s \cdot C} \cdot \frac{1}{R + \frac{1}{s \cdot C} + s \cdot L}$$

$$H(s) = \frac{1}{L \cdot C \cdot s^2 + R \cdot C \cdot s + 1}$$

2. Dynamic model in form state space representation equations.

General form of state space representation equations is following:

$$\dot{x} = [A] \cdot x + [B] \cdot u$$

$$y = [C] \cdot x + [D] \cdot u$$

Where:

[A]- state matrix, [B]- input matrix, [C]- output matrix, [D]- feedthrough matrix

We start calculating state space representation equations by writing Kirchhoff’s voltage law (KVL) equation for circuit.

$$u_1(t) - i(t) \cdot R - u_c(t) - u_L(t) = 0$$

$$u_1(t) - i(t) \cdot R - u_c(t) - u_2(t) = 0$$

$$i_c(t) = C \cdot \frac{du_c(t)}{dt}, \quad u_c(t) = L \cdot \frac{di_L(t)}{dt}$$

Because elements $R, L, C$ are connected in series then:

$$i(t) = i_c(t) = i_L(t)$$

$$u_1(t) - R \cdot C \cdot \frac{du_c(t)}{dt} - u_c(t) - L \cdot \frac{di(t)}{dt} = 0$$

$$u_1(t) - R \cdot C \cdot \frac{du_c(t)}{dt} - u_c(t) - L \cdot C \cdot \frac{d^2 u_c(t)}{dt^2} = 0$$

We write equation in this way to place derivative of the biggest degree at the left side of equation.
State space representation, state matrix [A] and input matrix [B]:

\[
\begin{bmatrix}
\frac{du_c(t)}{dt} \\
\frac{d^2u_c(t)}{dt^2}
\end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{L \cdot C} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} u_c(t) \\ \frac{du_c(t)}{dt} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L \cdot C} \end{bmatrix} \cdot [u_1(t)]
\]

State space representation, output matrix(vector) [C] and feedthrough matrix(vector) [D]:

\[
[u_c(t)] = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} u_c(t) \\ \frac{du_c(t)}{dt} \end{bmatrix} + [0] \cdot [u_1(t)]
\]

3. Simulations in MATLAB.
We have designated two dynamic models. First model is in form of transfer function. Second model is in form of state space representation equations. We have all necessary data to execute simulation in MATLAB.

3.1. Simulation in MATLAB through transfer function \(H(s)\).
Simulation of system answer for jump extortion it’s possible through \(lsim\) function. First we have to create m-file which contains parameters of electric circuit.

\(lsim(c, d, uin, t)\)

\textit{where:}
\begin{itemize}
  \item \textit{c} – vector with factors from counter of transfer function \(H(s)\)
  \item \textit{d} – vector with factors from denominator of transfer function \(H(s)\)
  \item \textit{uin} – vector with values for extortion
  \item \textit{t} – time vector
\end{itemize}

Sample code to executing simulation:

R=100;%resistance value
L=0.01;%inductivity value
C=0.001;%capacity value
t=0:0.000001:1;%time vector definition
uin=ones(1,length(t));%definition for unit extortion with application of ones function
c=[1];%vector with transfer function counter factors
d=[L*C R*C 1]; % vector with transfer function denominator factors
lsim(c,d,u_in,t);

3.2. Simulation in MATLAB through state space representation.
MATLAB is really very powerful calculation tool. It has in its resources functions which allow
us to plot impulse response characteristic, step response characteristic and frequency
response characteristics.

impulse(A,B,C,D)
step(A,B,C,D)
bode(A,B,C,D)

Using specified above functions we can plot characteristics of object. It is necessary to give
input parameters as: state matrix, input matrix, output matrix and feedthrough matrix. In
considered example appearance of input parameters is following:

\[
A = \begin{bmatrix}
0 & 1 \\
-\frac{1}{L \cdot C} & -\frac{R}{L}
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0 \\
\frac{1}{L \cdot C}
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
1 \\
0
\end{bmatrix}
\]

\[
D = [0]
\]

MATLAB gives us possibility to create our own functions. Plenty of functions which are in
MATLAB’s library were defined by users.