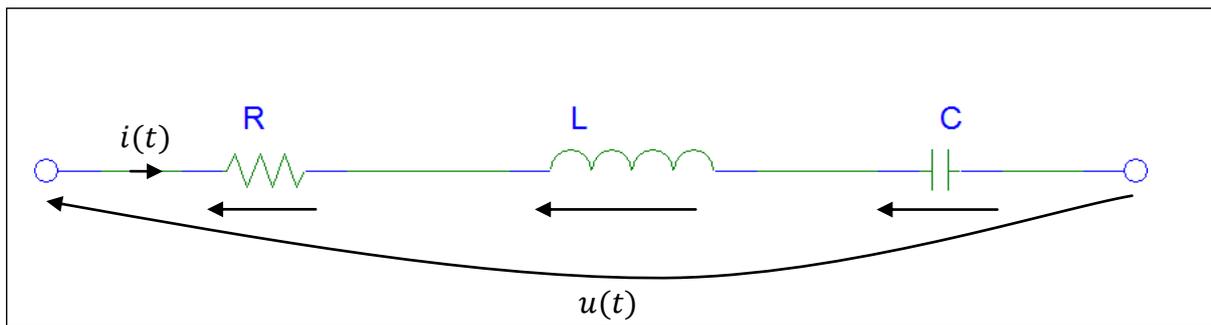


Impedance and reactance in two terminal RLC circuit

Two-terminal *RLC* circuit is built from resistor, capacitor and inductive. Between circuit terminal is alternating voltage $u(t)$. As an effect of voltage in circuit flows sine wave alternating current $i(t)$.



Through circuit flows alternate sine wave electric current described by equation:

$$i(t) = \sqrt{2} \cdot I \cdot \sin \omega \cdot t$$

Where:

$\sqrt{2} \cdot I$ – amplitude

I – root mean square value (RMS value)

ω – pulsation

f – frequency

Formulas which we will use to designate voltages on circuit's elements are following:

$$u_L = L \cdot \frac{di(t)}{dt}$$

$$u_C = \frac{1}{C} \cdot \int i \cdot dt$$

$$\omega = 2 \cdot \pi \cdot f$$

All elements in circuit are connected in series. Characteristic of series circuit is that the current which flows in it the same on all elements. Voltages on elements in circuit we will designate using formulas mentioned on previous page. We will use Ohm's law to designate voltage on resistor.

Voltage on resistor(Ohm's law):

$$u_R = R \cdot i(t)$$

Voltage on inductance:

$$u_L = L \cdot \frac{di(t)}{dt} = L \cdot \frac{d}{dt}(\sqrt{2} \cdot I \cdot \sin \omega \cdot t) = \omega \cdot L \cdot \sqrt{2} \cdot I \cdot \cos \omega \cdot t$$

At the start of calculated derivative appeared factor $\omega \cdot L$, it is inductive reactance.

$$X_L = \omega \cdot L$$

Taking into consideration factor describing inductive reactance:

$$u_L = X_L \cdot \sqrt{2} \cdot I \cdot \cos \omega \cdot t$$

You can note that voltage on inductivity overtakes in phase current which flows through inductivity and circuit. Voltage on inductivity we can write in form:

$$u_L = X_L \cdot \sqrt{2} \cdot I \cdot \sin \omega \cdot t + \frac{\pi}{2}$$

Voltage on capacitor:

$$\begin{aligned} u_C &= \frac{1}{C} \cdot \int_0^t i \cdot dt = \frac{1}{C} \cdot \int_0^t (\sqrt{2} \cdot I \cdot \sin \omega \cdot t) \cdot dt \\ &= -\frac{1}{\omega \cdot C} \cdot \sqrt{2} \cdot I \cdot \cos \omega \cdot t \end{aligned}$$

After calculating definite integral, we have received a factor $1/(\omega \cdot C)$, it is capacitive reactance. As you can note voltage on capacitor is described by function $-\cos \omega \cdot t$ which is multiplied only by constants. From trigonometry we have relation $-\cos \omega \cdot t = \sin \omega \cdot t - \frac{\pi}{2}$, so expression which describes voltage on capacitor we can write as:

$$u_C = -X_C \cdot \sqrt{2} \cdot I \cdot \cos \omega \cdot t$$

You can note that voltage on capacitor is late in phase in relation to current which flows through capacitor and circuit.

$$u_C = \frac{1}{\omega \cdot C} \cdot \sqrt{2} \cdot I \cdot \sin \omega \cdot t - \frac{\pi}{2}$$

$$u_C = X_C \cdot \sqrt{2} \cdot I \cdot \sin \omega \cdot t - \frac{\pi}{2}$$

General formulas are designated, now we will consider simulation in MATLAB. We assume that component elements of circuit are ideal elements. It means that they don't have any parasitic parameters. For example in reality it is impossible to find ideal element. For example real resistor has parasitic capacity and parasitic inductivity which in relation to resistance depends from frequency. For low frequencies we can omit that parasitic parameters. The same case regards to ideal inductivity and ideal capacitor. Elements in considered circuit we treat as elements with focused parameters.

Parameters for simulation are following:

$$I = 1[A] - \text{current RMS value}$$

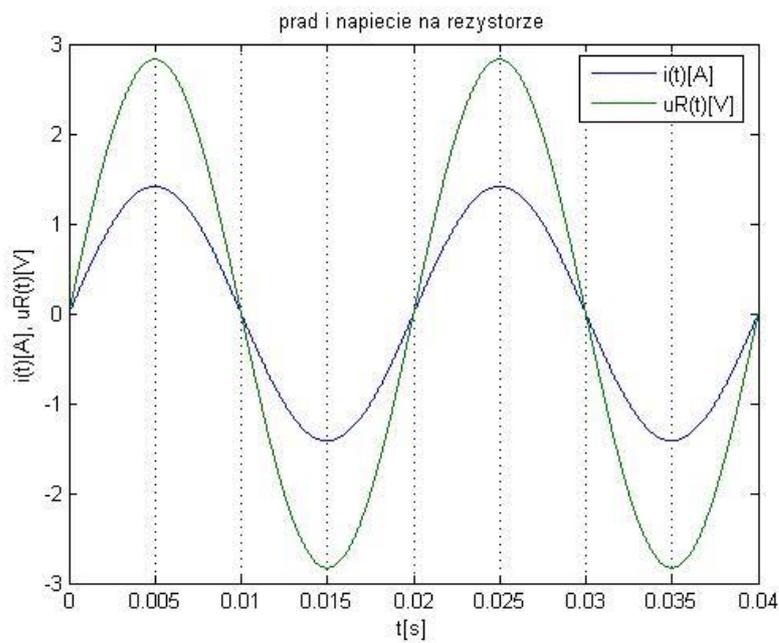
$$f = 50[Hz]$$

$$R = 2[\Omega]$$

$$L = 3,1831[mH] \rightarrow X_L = 1[\Omega]$$

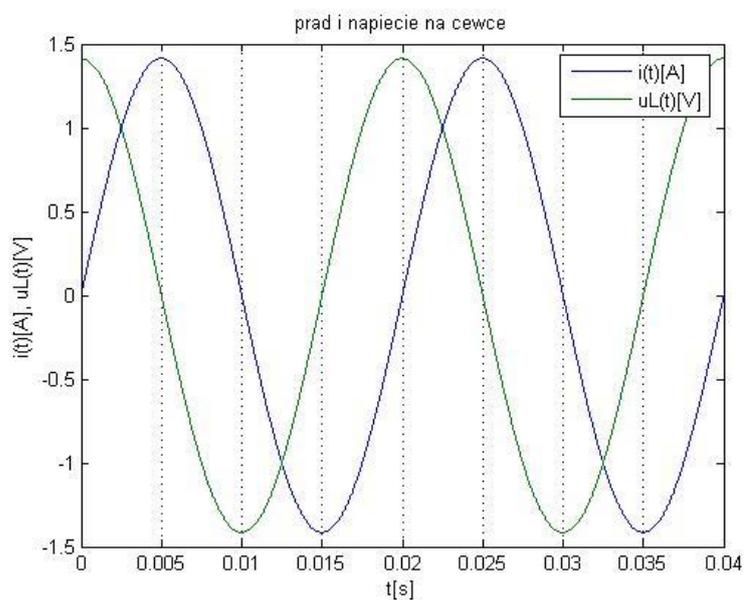
$$C = 3,1831[mF] \rightarrow X_C = 1[\Omega]$$

As you can see values of capacitive reactance and inductive reactance are equal. It means that it is specific situation. Situation when inductive reactance and capacitive reactance are equal is called resonance.



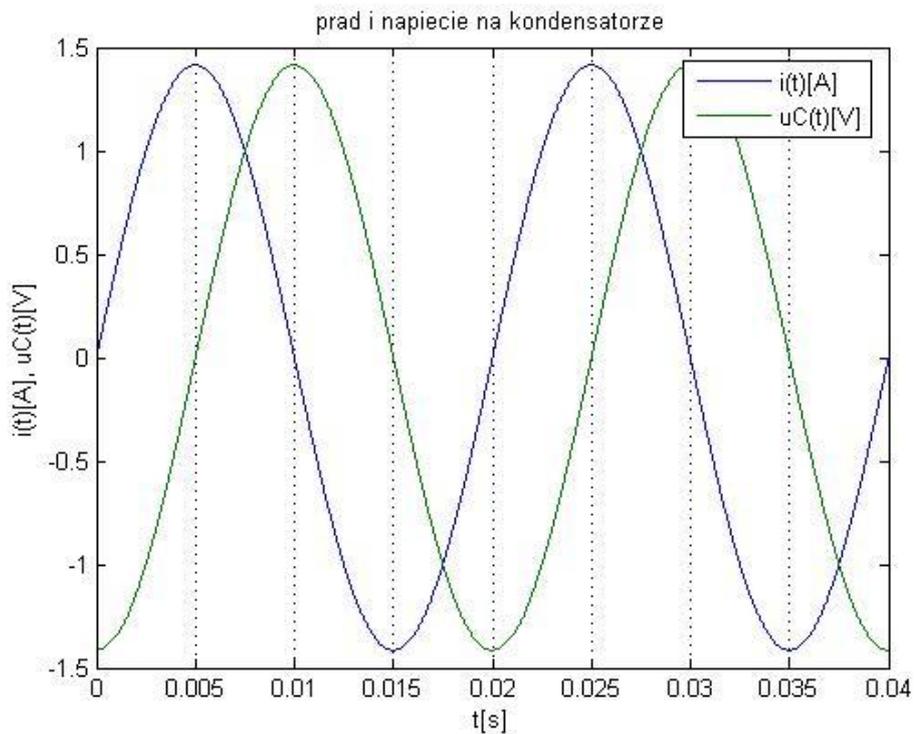
Picture 1. Current and voltage waveform of resistor R.

Picture placed above shows waveforms of resistor's voltage and current. You can note that voltage and current on resistor are equal in phase.



Picture 2. Current and voltage waveform of inductivity L.

On picture 2 waveforms of current and voltage on inductivity L are placed. You can note that voltage overtakes in phase current by angle $\frac{\pi}{2}$.



Picture 3. Current and voltage waveform of capacitor C.

On picture 2 waveforms of current and voltage on capacitor C are placed. You can note that voltage lags in phase to current by angle $\frac{\pi}{2}$.

Three plots placed above contain waveforms of voltages and currents of elements which create circuit. Electric circuit in example is serial electric circuit. Characteristic of serial circuit is that the same current flows through whole circuit. Value and shape of electric current $i(t)$ is the same for all elements in circuit.

Impedance is in general a vector. Vector of impedance contains three components: resistance, inductive reactance and capacitive reactance. Vector of impedance is geometric sum these components.

$$\underline{Z} = R + j \cdot X_L - j \cdot X_C$$

$$\underline{Z} = R + j \cdot \omega \cdot L - j \cdot \frac{1}{\omega \cdot C}$$

$$\underline{Z} = R + j \cdot \omega \cdot L + \frac{1}{j \cdot \omega \cdot C}$$

Where $j^2 = -1$ is an imaginary unit

$$-j = \frac{1}{j} \text{ and } j = \frac{1}{-j}$$

Because components of impedance are summing geometrical. Value of impedance is described by formula:

$$|\underline{Z}| = \sqrt{R^2 + (X_L - X_C)^2}$$



Picture 4. Vector diagram of impedance components.

Circuit taken under consideration in example has for frequency $f = 50[\text{Hz}]$ equal values of inductive reactance X_L and capacitive reactance X_C . Both for $f = 50[\text{Hz}]$ have value $X_L = X_C = 1[\Omega]$. Electric circuit is in current resonance because both reactances compensate each other. In this specific case value of current which flows through circuit is the biggest. This phenomena is used in practice for example in selective filters, where for wide spectrum of frequencies current's value is low but for thin part of frequencies spectrum current's value is a lot of bigger.

$$|\underline{Z}| = R = 10[\Omega]$$