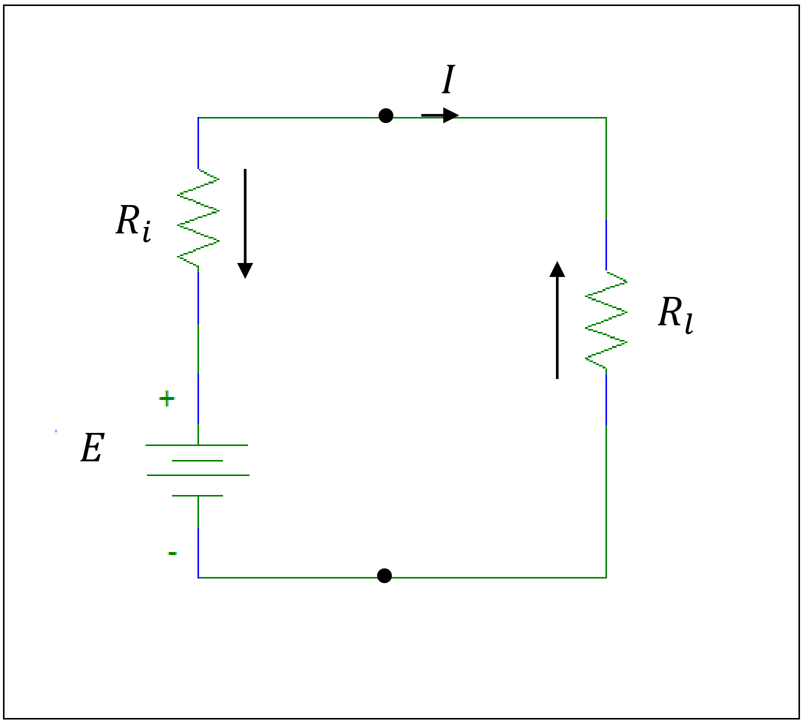
http://www.mbstudent.com/electrical-engineering.html

Designation of maximum power on load resistor.

Our task in this example is to designate maximum power on load resistor. Electric circuit is built from voltage source E, which has internal resistance R_i. To voltage source terminals is connected load resistor R_l. All elements in circuit are connected in series. We treat elements in circuit like elements with focused parameters. We assume that resistance of connection wires is very low and we can omit it.



We write Kirchhoff's voltage law (KVL) equation for circuit.

$$E - R_i \cdot I - R_l \cdot I = 0$$

Current I which flows through circuit is given by formula:

$$I = \frac{E}{R_i + R_l}$$

Generally power in DC circuit is given by formula:

$$P = U \cdot I$$

Using Ohm's law which allow us to write voltage as function of current. We receive:

$$U = R \cdot I$$

After taking under consideration expression for voltage, formula for power is following:

$$P = I^2 \cdot R$$

General formulas are known. We will write equation for power which is released on load resistor. Resistor R_l is load resistor.

$$P(R_l) = I^2 \cdot R_l$$

$$P(R_l) = \left(\frac{E}{R_w + R_l}\right)^2 \cdot R_l$$

Subject of example is to find value resistor R_l for which power on it will be have maximum value. To designate that specific value of resistor R_l we have to calculate a derivative of power P in function of resistor R_l.

$$\frac{dP(R_l)}{dR_l} = \left(\frac{E}{R_i + R_l}\right)^2 \cdot \frac{d}{dR_l} R_l + R_l \cdot \frac{d}{dR_l}\left(\left(\frac{E}{R_i + R_l}\right)^2\right)$$

$$\frac{dP(R_l)}{dR_l} = \left(\frac{E}{R_i + R_l}\right)^2 + 2 \cdot \frac{E}{R_i + R_l} \cdot (-1) \cdot \frac{E}{(R_i + R_l)^2}$$

$$\frac{dP(R_l)}{dR_l} = \left(\frac{E}{R_i + R_l}\right)^2 - 2 \cdot R_l \cdot \frac{E}{R_i + R_l} \cdot \frac{E}{(R_i + R_l)^2}$$

$$\frac{dP(R_l)}{dR_l} = \left(\frac{E}{R_i + R_l}\right)^2 - 2 \cdot R_l \cdot \frac{E^2}{(R_i + R_l)^3}$$

To find value of resistor R_l for which function $P(R_l)$ has maximum value we have to equate first derivative of function $P(R_l)$ to zero.

$$\frac{dP(R_l)}{dR_l} = 0$$

$$0 = \left(\frac{E}{R_i + R_l}\right)^2 - 2 \cdot R_l \cdot \frac{E^2}{(R_i + R_l)^3}$$

$$\left(\frac{E}{R_i + R_l}\right)^2 = 2 \cdot R_l \cdot \frac{E^2}{(R_i + R_l)^3}$$

$$\frac{E^2}{(R_i + R_l)^2} = 2 \cdot R_l \cdot \frac{E^2}{(R_i + R_l)^3}$$

Next we multiply both sides of equation by the same factor.

$$\frac{E^2}{(R_i + R_l)^2} \cdot \frac{(R_i + R_l)^2}{E^2} = 2 \cdot R_l \cdot \frac{E^2}{(R_i + R_l)^3} \cdot \frac{(R_i + R_l)^2}{E^2}$$

$$1 = 2 \cdot R_l \cdot \frac{1}{R_i + R_l}$$

$$R_i + R_l = 2 \cdot R_l$$

$$R_i = 2 \cdot R_l - R_l$$

$$R_l = R_i$$

And finally we have a result which we were looking for. Correctly with formula derivation conclude is as follows. Maximum power will be released on load resistor when its value is equal to value of voltage source internal resistance. Load resistance has to be matched to source resistance. The same conclude we can form in another way. To receive maximum power on output, output resistance has to be equal to input resistance.

$$P(R_l) = \left(\frac{E}{R_i + R_l}\right)^2 \cdot R_l$$

Because

$$R_l = R_i$$

Then

$$P_{max} = \left(\frac{E}{R_i + R_i}\right)^2 \cdot R_i$$

$$P_{max} = \left(\frac{E}{2 \cdot R_i}\right)^2 \cdot R_i$$

$$P_{max} = \frac{E^2}{4 \cdot {R_i}^2} \cdot R_i$$

Finally the maximum power which is possible to receive on output is equal to:

$$P_{max} = \frac{E^2}{4 \cdot R_i}$$