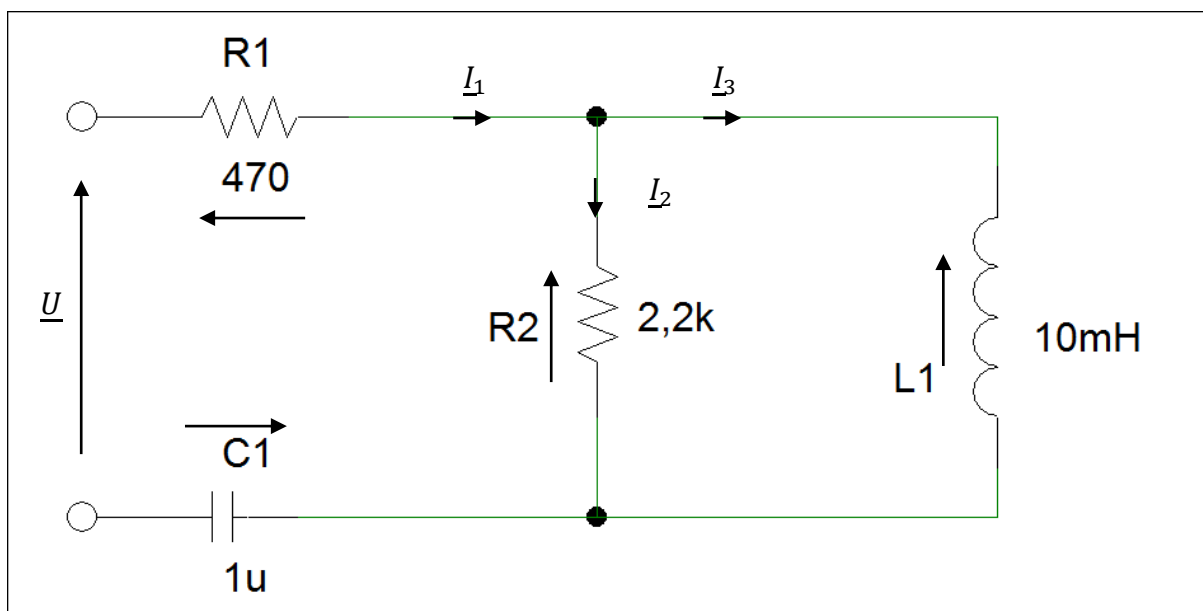


Total impedance and vector diagrams in AC electric circuit.

Purpose of this example is to designate a total impedance of electric circuit seen from two chosen terminals. Additionally vector diagrams of currents and voltage in following branches of circuit will be written. Remember about characteristics of series and parallel connections:

1. *series connection* → *the same current*
2. *parallel connection* → *the same voltage*

Obviously characteristics above regard to elements with focus parameters.



Picture 1. Example AC electric circuit.

Note that in general impedance is a vector which has two components:

- *resistance* R
- *inductive reactance* $X_L = \omega \cdot L$;
- *capacity reactance* $X_C = \frac{1}{\omega \cdot C}$
- *pulsation* $\omega = 2 \cdot \pi \cdot f$

Where j is a imaginary unit. From complex numbers branch in mathematics you should also know that $j^2 = -1$.

$$\underline{Z} = R + j \cdot \omega \cdot L - j \cdot \frac{1}{\omega \cdot C}$$

$$\underline{Z} = R + j \cdot X_L - j \cdot X_C$$

As you have noted inductive reactance X_L and capacity reactance X_C subtract each other. It is possible that this both reactance will compensate. That specific case in which reactances compensate it is called electric resonance.

$$\underline{Z} = R + j \cdot X$$

$$\text{where } X = X_L - X_C$$

Impedance dimension is Ohm [Ω]. The same dimension have resistance and reactance.

Impedance is tied with another very useful magnitude which is admittance. Impedance and admittance are tied in simple formula:

$$\underline{Z} = \frac{1}{\underline{Y}}$$

$$\underline{Y} = \frac{1}{\underline{Z}}$$

Admittance is a vector which contains two components

- *conductance* G
- *inductive susceptance* $B_L = \frac{1}{\omega \cdot L}$
- *capacity susceptance* $B_C = \omega \cdot C$

$$\underline{Y} = G + j \cdot \omega \cdot C - j \cdot \frac{1}{\omega \cdot L}$$

$$\underline{Y} = G + j \cdot B_C - j \cdot B_L$$

$$\underline{Y} = G + j \cdot B$$

$$\text{where } B = B_C - B_L$$

Admittance dimension is Siemens [S]. The same dimension have conductance and susceptance.

We will start designating total impedance from last elements in circuit. These elements are inductivity L1 and resistor R2. That elements are connected in parallel. We will use formula for admittance.

$$\begin{aligned}\underline{Z}_{R2} &= R2 \rightarrow \underline{Y}_{R2} = \frac{1}{R2} \\ \underline{Z}_{L1} &= j \cdot \omega \cdot L1 \rightarrow \underline{Y}_{L1} = \frac{1}{j \cdot \omega \cdot L1} \\ \underline{Y}_{R2L1} &= \underline{Y}_{R2} + \underline{Y}_{L1} = \frac{1}{R2} + \frac{1}{j \cdot \omega \cdot L1} \\ \underline{Y}_{R2L1} &= \frac{j \cdot \omega \cdot L1}{R2 \cdot j \cdot \omega \cdot L1} + \frac{R2}{R2 \cdot j \cdot \omega \cdot L1} = \frac{R2 + j \cdot \omega \cdot L1}{R2 \cdot j \cdot \omega \cdot L1}\end{aligned}$$

$$\begin{aligned}\underline{Z}_{R2L1} &= \underline{Y}_{R2L1}^{-1} \\ \underline{Z}_{R2L1} &= \frac{R2 \cdot j \cdot \omega \cdot L1}{R2 + j \cdot \omega \cdot L1}\end{aligned}$$

For future calculations we have to multiply counter and denominator by expression which is coupled to complex number in denominator. This method will remove complex number from denominator.

$$\begin{aligned}\underline{Z}_{R2L1} &= \frac{R2 \cdot j \cdot \omega \cdot L1}{R2 + j \cdot \omega \cdot L1} \cdot \frac{R2 - j \cdot \omega \cdot L1}{R2 - j \cdot \omega \cdot L1} \\ \underline{Z}_{R2L1} &= \frac{R2^2 \cdot j \cdot \omega \cdot L1 + R2 \cdot \omega^2 \cdot L1^2}{R2^2 - R2 \cdot j \cdot \omega \cdot L1 + R2 \cdot j \cdot \omega \cdot L1 + \omega^2 \cdot L1^2} \\ \underline{Z}_{R2L1} &= \frac{R2 \cdot \omega^2 \cdot L1^2 + R2^2 \cdot j \cdot \omega \cdot L1}{R2^2 + \omega^2 \cdot L1^2} \\ \underline{Z}_{R2L1} &= \frac{R2 \cdot \omega^2 \cdot L1^2}{R2^2 + \omega^2 \cdot L1^2} + j \cdot \frac{R2^2 \cdot \omega \cdot L1}{R2^2 + \omega^2 \cdot L1^2}\end{aligned}$$

Component impedance \underline{Z}_{R2L1} is in series connected with impedances \underline{Z}_{R1} and \underline{Z}_{C1} . Total impedance of circuit which is seen from input terminals is given by equation:

$$\underline{Z}_t = \underline{Z}_{R1} + \underline{Z}_{R2L1} + \underline{Z}_{C1}$$

$$\underline{Z}_t = R1 + \frac{R2 \cdot \omega^2 \cdot L1^2}{R2^2 + \omega^2 \cdot L1^2} + j \cdot \frac{R2^2 \cdot \omega \cdot L1}{R2^2 + \omega^2 \cdot L1^2} - j \cdot \frac{1}{\omega \cdot C1}$$

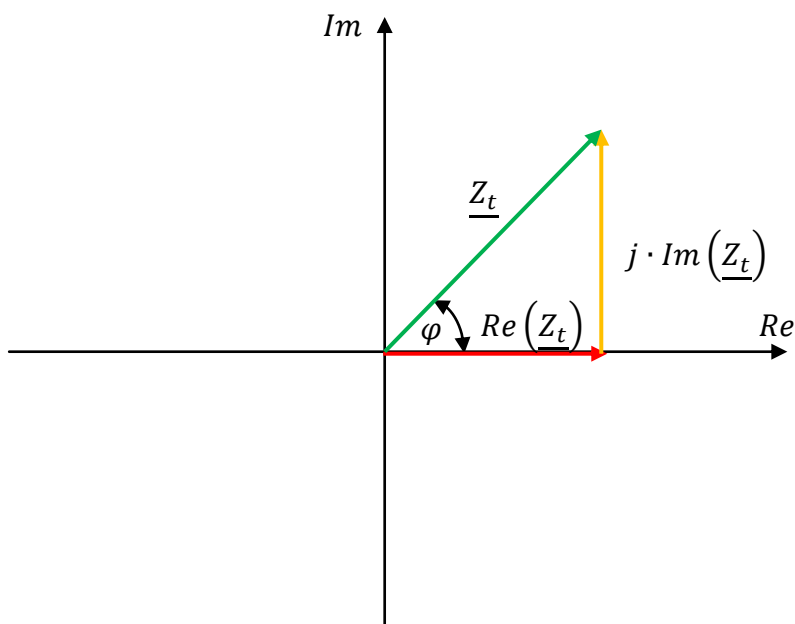
$$\underline{Z}_t = R1 + \frac{R2 \cdot \omega^2 \cdot L1^2}{R2^2 + \omega^2 \cdot L1^2} + j \cdot \left(\frac{R2^2 \cdot \omega \cdot L1}{R2^2 + \omega^2 \cdot L1^2} - \frac{1}{\omega \cdot C1} \right)$$

Because impedance is complex number its value is given by formula:

$$|\underline{Z}_t| = \sqrt{Re(\underline{Z}_t)^2 + Im(\underline{Z}_t)^2}$$

$$|\underline{Z}_t| = \sqrt{\left(R1 + \frac{R2 \cdot \omega^2 \cdot L1^2}{R2^2 + \omega^2 \cdot L1^2} \right)^2 + \left(\frac{R2^2 \cdot \omega \cdot L1}{R2^2 + \omega^2 \cdot L1^2} - \frac{1}{\omega \cdot C1} \right)^2}$$

Note that imaginary part of complex impedance is total reactance of circuit $Im(\underline{Z}_t) = X$.



Picture 2. Total impedance's vector and its components on complex variable plane.

Note that the lengths of vectors depend from parameters values in complex number. On picture is shown specific situation when imaginary part is equal reality part of total impedance's vector.

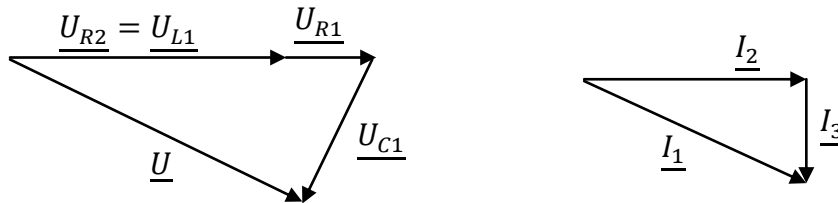
Total impedance of this circuit can be written in another complex number form

$$\underline{Z}_t = Z_t \cdot e^{j \cdot \varphi}$$

Total impedance's value and vector is designated.

Now vectors of currents and voltages in circuit will be plotted. Note that lengths of vectors depend from their values. We have formulas only with symbols. We don't calculate numeric value of vectors lengths.

We will start plotting vectors from last elements in circuit. These elements are inductivity $L1$ and resistor $R2$. Because that elements are connected in parallel they have the same voltage. Next we have to add voltages on resistor $R1$ and capacitor $C1$.



Picture 2. Vector diagram of voltages and currents in circuit.

Note that shape of voltages and current triangles depend from values of currents and voltages in circuit. We didn't calculate values. If you will calculate values shape of real triangles could differ. Remember that voltage $\underline{U_{C1}}$ on capacitor $C1$ lags in phase by $\frac{\pi}{2}$ to current $\underline{I_1}$.