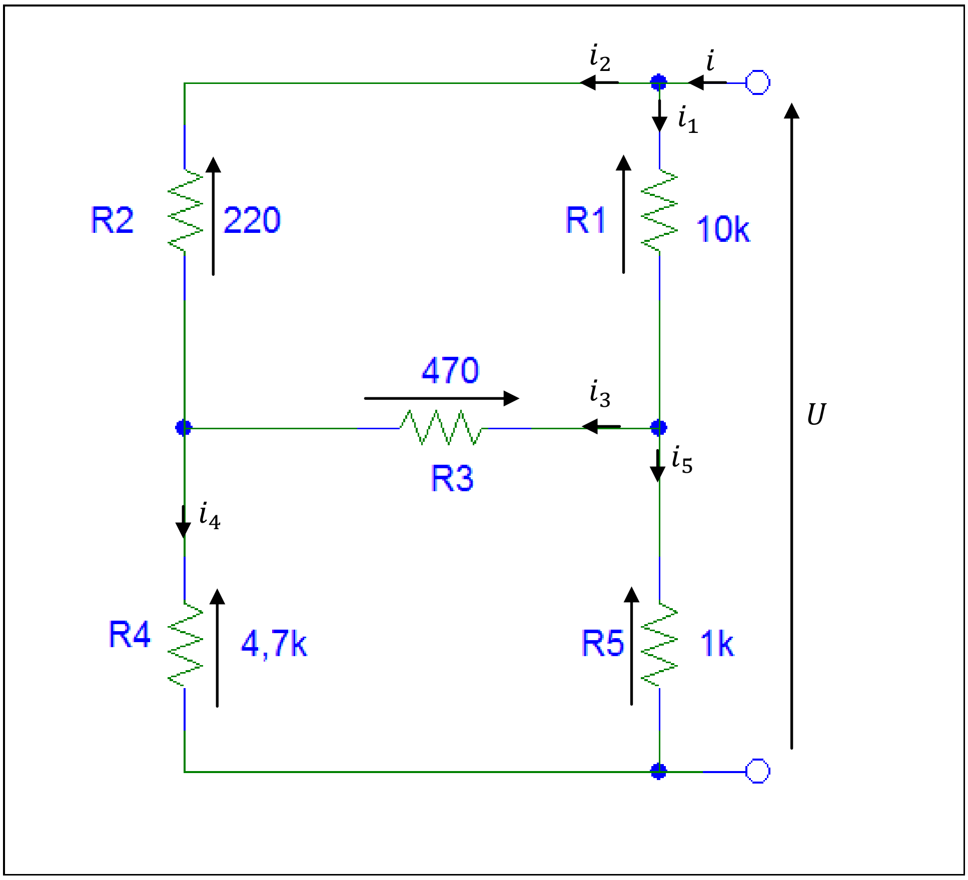
http://www.mbstudent.com/electrical-engineering.html

Total resistance of electric DC circuit.

At first look, circuit below looks complicated to calculate its total resistance R from A and B terminals. We will use branch current method to calculate total resistance R of this circuit. Resistors are connected in specific way because they create H bridge.



Picture 1. Example electric circuit with resistor connected to H bridge.

If we connect voltage U to A and B terminals of circuit, current i will flow through circuit.

[0] $$i = \frac{U}{R}$$

Kirchhoff's current law (KCL) equations for all nodes in circuit.

[1] $$i - i_1 - i_2 = 0$$

[2] $$i_1 - i_3 - i_5 = 0$$

[3] $$i_2 + i_3 - i_4 = 0$$

[4] $$i_4 + i_5 - i = 0$$

Kirchhoff's voltage law (KVL) equations for all meshes.

[5] $$-i_3 \cdot R3 + i_2 \cdot R2 - i_1 \cdot R1 = 0$$

[6] $$i_4 \cdot R4 + i_3 \cdot R3 - i_5 \cdot R5 = 0$$

[7] $$i_5 \cdot R5 + i_1 \cdot R1 - U = 0$$

[8] $$i_2 \cdot R2 + i_4 \cdot R4 - U = 0$$

We will add equations [7] and [8].

$$i_5 \cdot R5 + i_1 \cdot R1 - U + i_2 \cdot R2 + i_4 \cdot R4 - U = 0$$

$$i_5 \cdot R5 + i_1 \cdot R1 + i_2 \cdot R2 + i_4 \cdot R4 = 2 \cdot U$$

We will use Kirchhoff's current law equations [2] and [3]. These equations will help us in eliminating currents i_4 and i_5 from equation.

$$i_5 = i_1 - i_3$$

$$i_4 = i_2 + i_3$$

$$(i_1 - i_3) \cdot R5 + i_1 \cdot R1 + i_2 \cdot R2 + (i_2 + i_3) \cdot R4 = 2 \cdot U$$

$$i_1 \cdot R5 - i_3 \cdot R5 + i_1 \cdot R1 + i_2 \cdot R2 + i_2 \cdot R4 + i_3 \cdot R4 = 2 \cdot U$$

$$i_1 \cdot (R1 + R5) + i_2 \cdot (R2 + R4) + i_3 \cdot (R4 - R5) = 2 \cdot U$$

In equation above we have to eliminate current i_3. We will use equation [5] to do this.

$$-i_3 \cdot R3 + i_2 \cdot R2 - i_1 \cdot R1 = 0$$

$$i_3 = \frac{i_2 \cdot R2 - i_1 \cdot R1}{R3}$$

$$i_1 \cdot (R1 + R5) + i_2 \cdot (R2 + R4) + \frac{i_2 \cdot R2 - i_1 \cdot R1}{R3} \cdot (R4 - R5) = 2 \cdot U$$

$$i_1 \cdot (R1 + R5) + i_2 \cdot (R2 + R4) + \frac{i_2 \cdot R2 \cdot (R4 - R5)}{R3} - \frac{i_1 \cdot R1 \cdot (R4 - R5)}{R3} = 2 \cdot U$$

$$i_1 \cdot \left(R1 + R5 - \frac{R1 \cdot (R4 - R5)}{R3}\right) + i_2 \cdot \left(R2 + R4 + \frac{R2 \cdot (R4 - R5)}{R3}\right) = 2 \cdot U$$

Factors which are multiplied by currents i_1 and i_2 contains only constants. We will assign to them new symbols to make equation easier to write.

$$i_1 \cdot A_R + i_2 \cdot B_R = 2 \cdot U$$

$$A_R = R1 + R5 - \frac{R1 \cdot (R4 - R5)}{R3}$$

$$B_R = R2 + R4 + \frac{R2 \cdot (R4 - R5)}{R3}$$

At this point we have to eliminate current i_2 from equation. We will use equation [1].

$$i - i_1 - i_2 = 0$$

$$i_2 = i - i_1$$

$$i_1 \cdot A_R + (i - i_1) \cdot B_R = 2 \cdot U$$

$$i_1 \cdot A_R + i \cdot B_R - i_1 \cdot B_R = 2 \cdot U$$

$$i_1 \cdot (A_R - B_R) = 2 \cdot U - i \cdot B_R$$

Because current i is tied with supply voltage U by Ohm's law, we will replace it in equation.

$$i_1 \cdot (A_R - B_R) = 2 \cdot U - \frac{U}{R} \cdot B_R$$

$$i_1 \cdot (A_R - B_R) = U \cdot \left(2 - \frac{B_R}{R}\right)$$

[*] $$i_1 = U \cdot \left(2 - \frac{B_R}{R}\right) \cdot (A_R - B_R)^{-1}$$

We will add equations [7] and [8].

$$i_5 \cdot R5 + i_1 \cdot R1 - U + i_2 \cdot R2 + i_4 \cdot R4 - U = 0$$

$$i_5 \cdot R5 + i_1 \cdot R1 + i_2 \cdot R2 + i_4 \cdot R4 = 2 \cdot U$$

We will use Kirchhoff's current law equations [2] and [3]. These equations will help us in eliminating currents i_1 and i_2 from equation.

$$i_1 = i_3 + i_5$$

$$i_2 = i_4 - i_3$$

$$i_5 \cdot R5 + (i_3 + i_5) \cdot R1 + (i_4 - i_3) \cdot R2 + i_4 \cdot R4 = 2 \cdot U$$

$$i_5 \cdot R5 + i_3 \cdot R1 + i_5 \cdot R1 + i_4 \cdot R2 - i_3 \cdot R2 + i_4 \cdot R4 = 2 \cdot U$$

$$i_5 \cdot (R1 + R5) + i_4 \cdot (R2 + R4) + i_3 \cdot (R1 - R2) = 2 \cdot U$$

In equation above we have to eliminate current i_3. We will use equation [6] to do this.

$$i_4 \cdot R4 + i_3 \cdot R3 - i_5 \cdot R5 = 0$$

$$i_3 \cdot R3 = i_5 \cdot R5 - i_4 \cdot R4$$

$$i_3 = \frac{i_5 \cdot R5 - i_4 \cdot R4}{R3}$$

$$i_5 \cdot (R1 + R5) + i_4 \cdot (R2 + R4) + \frac{i_5 \cdot R5 - i_4 \cdot R4}{R3} \cdot (R1 - R2) = 2 \cdot U$$

$$i_5 \cdot (R1 + R5) + i_4 \cdot (R2 + R4) + \frac{i_5 \cdot R5 \cdot (R1 - R2)}{R3} - \frac{i_4 \cdot R4 \cdot (R1 - R2)}{R3} = 2 \cdot U$$

$$i_5 \cdot \left(R1 + R5 + \frac{R5 \cdot (R1 - R2)}{R3}\right) + i_4 \cdot \left(R2 + R4 - \frac{R4 \cdot (R1 - R2)}{R3}\right) = 2 \cdot U$$

Factors which are multiplied by currents i_5 and i_4 contains only constants. We will assign to them new symbols to make equation easier to write.

$$i_5 \cdot C_R + i_4 \cdot D_R = 2 \cdot U$$

$$C_R = R1 + R5 + \frac{R5 \cdot (R1 - R2)}{R3}$$

$$D_R = R2 + R4 - \frac{R4 \cdot (R1 - R2)}{R3}$$

At this point we have to eliminate current i_4 from equation. We will use equation [4].

$$i_4 + i_5 - i = 0$$

$$i_4 = i - i_5$$

$$i_5 \cdot C_R + (i - i_5) \cdot D_R = 2 \cdot U$$

$$i_5 \cdot C_R + i \cdot D_R - i_5 \cdot D_R = 2 \cdot U$$

$$i_5 \cdot (C_R - D_R) = 2 \cdot U - i \cdot D_R$$

Because current i is tied with supply voltage U by Ohm's law, we will replace it in equation.

$$i_5 \cdot (C_R - D_R) = 2 \cdot U - \frac{U}{R} \cdot D_R$$

$$i_5 \cdot (C_R - D_R) = U \cdot \left(2 - \frac{D_R}{R}\right)$$

[**] $$i_5 = U \cdot \left(2 - \frac{D_R}{R}\right) \cdot (C_R - D_R)^{-1}$$

Now we will use equations for current $i_1 \rightarrow [*]$ and $i_5 \rightarrow [**]$. We will use equation [7] to derive total resistance R.

$$i_5 \cdot R5 + i_1 \cdot R1 = U$$

$$U \cdot \left(2 - \frac{D_R}{R}\right) \cdot (C_R - D_R)^{-1} \cdot R5 + U \cdot \left(2 - \frac{B_R}{R}\right) \cdot (A_R - B_R)^{-1} \cdot R1 = U$$

Both sides of equation will be divided by voltage U.

$$\left(2 - \frac{B_R}{R}\right) \cdot (A_R - B_R)^{-1} \cdot R1 + \left(2 - \frac{D_R}{R}\right) \cdot (C_R - D_R)^{-1} \cdot R5 = 1$$

We will from left side of equation a fraction with the same denominator.

$$R1 \cdot \left(2 - \frac{B_R}{R}\right) \cdot \frac{(C_R - D_R)}{(A_R - B_R) \cdot (C_R - D_R)} + R5 \cdot \left(2 - \frac{D_R}{R}\right) \cdot \frac{(A_R - B_R)}{(A_R - B_R) \cdot (C_R - D_R)} = 1$$

$$R1 \cdot \left(2 - \frac{B_R}{R}\right) \cdot (C_R - D_R) + R5 \cdot \left(2 - \frac{D_R}{R}\right) \cdot (A_R - B_R) = (A_R - B_R) \cdot (C_R - D_R)$$

$$\left(2 \cdot R1 - \frac{R1 \cdot B_R}{R}\right) \cdot (C_R - D_R) + \left(2 \cdot R5 - \frac{R5 \cdot D_R}{R}\right) \cdot (A_R - B_R) = (A_R - B_R) \cdot (C_R - D_R)$$

$$2 \cdot R1 \cdot (C_R - D_R) - \frac{R1 \cdot B_R}{R} \cdot (C_R - D_R) + 2 \cdot R5 \cdot (A_R - B_R) - \frac{R5 \cdot D_R}{R} \cdot (A_R - B_R) = (A_R - B_R) \cdot (C_R - D_R)$$

$$-\frac{R1 \cdot B_R}{R} \cdot (C_R - D_R) - \frac{R5 \cdot D_R}{R} \cdot (A_R - B_R) = (A_R - B_R) \cdot (C_R - D_R) - 2 \cdot (R1 \cdot (C_R - D_R) + R5 \cdot (A_R - B_R))$$

$$R = \frac{-R1 \cdot B_R \cdot (C_R - D_R) - R5 \cdot D_R \cdot (A_R - B_R)}{(A_R - B_R) \cdot (C_R - D_R) - 2 \cdot \big(R1 \cdot (C_R - D_R) + R5 \cdot (A_R - B_R)\big)}$$

As you can see above total resistance of this circuit is described by really "big" fraction. You have to take under consideration expressions for constants A_R, B_R, C_R, D_R. You can check correction of this formula by simulation in Pspice program and by using total resistance calculation which I have made in Excel.