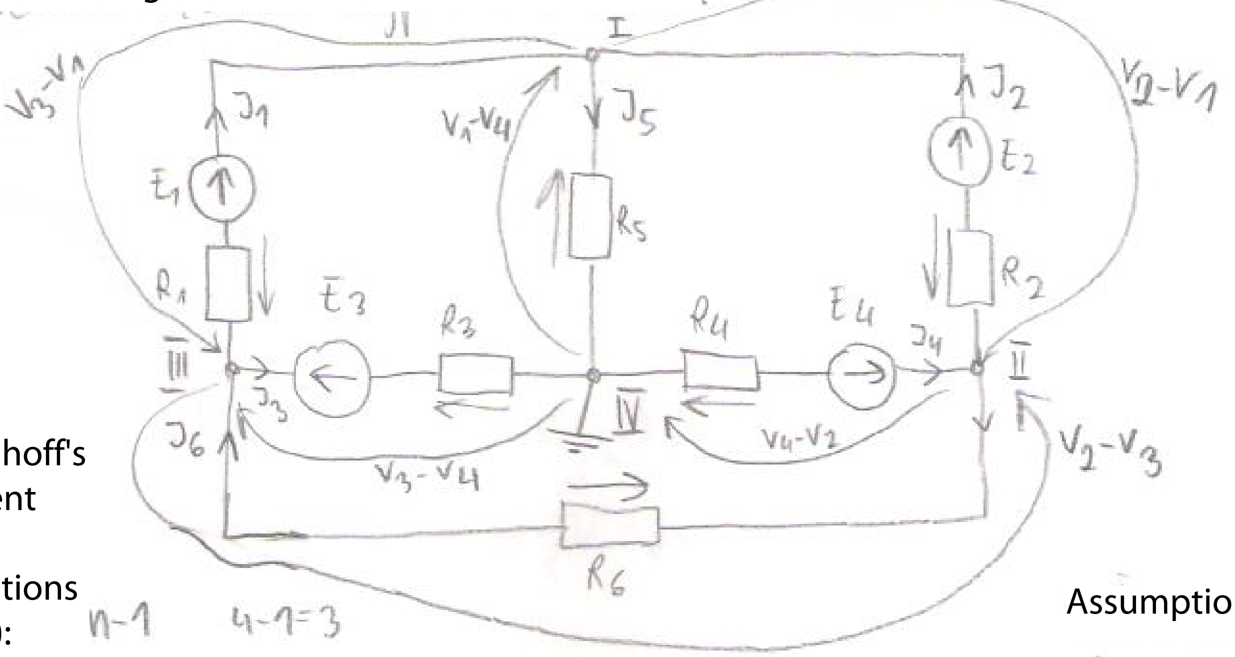
Node voltage method - derivation



Kirchhoff's current law equations (KCL):

n-1 4-1=3

Assumption:

$V_4=0$

Node I:

$$J_1+J_2-J_5=0$$

Node II:

$$J_4-J_2-J_6=0$$

Node III:

$$J_6-J_3-J_1=0$$

Kirchhoff's voltage law (KVL) equations for each branch separately:

$$(V_3-V_1)-J_1R_1+E_1=0 \Rightarrow J_1=\frac{(V_3-V_1)+E_1}{R_1} \Rightarrow J_1=(V_3-V_1)\cdot G_1+E_1\cdot G_1$$

$$(V_2-V_1)-J_2R_2+E_2=0 \Rightarrow J_2=\frac{(V_2-V_1)+E_2}{R_2} \Rightarrow J_2=(V_2-V_1)\cdot G_1+E_2G_2$$

$$(V_1-V_4)-J_5R_5=0 \Rightarrow J_5=\frac{(V_1-V_4)}{R_5} \Rightarrow J_5=(V_1-V_4)\cdot G_5$$

$$(V_3-V_4)-E_3-J_3R_3=0 \Rightarrow J_3=\frac{(V_3-V_4)-E_3}{R_3} \Rightarrow J_3=(V_3-V_4)\cdot G_3-E_3G_3$$

$$(V_4-V_2)-J_4R_4+E_4=0 \Rightarrow J_4=\frac{(V_4-V_2)+E_4}{R_4} \Rightarrow J_4=(V_4-V_2)\cdot G_4+E_4\cdot G_4$$

$$(V_2-V_3)-J_6R_6=0 \Rightarrow J_6=\frac{(V_2-V_3)}{R_6} \Rightarrow J_6=(V_2-V_3)\cdot G_6$$

Currents expressed by equations above are inserted to Kirchhoff's current law (KCL) equations:

$$\begin{cases} (V_3-V_1)\cdot G_1 + E_1\cdot G_1 + (V_2-V_1)\cdot G_2 + E_2 G_2 - (V_1-V_4)\cdot G_5 = 0 & \text{Node I} \\ (V_4-V_2)\cdot G_4 + E_4 G_4 - (V_2-V_1)\cdot G_2 - E_2 G_2 - (V_2-V_3)\cdot G_6 = 0 & \text{Node II} \\ (V_2-V_3)\cdot G_6 - (V_3-V_4)\cdot G_3 + E_3 G_3 - (V_3-V_1)\cdot G_1 - E_1\cdot G_1 = 0 & \text{Node III} \end{cases}$$

$$\begin{cases} V_3\cdot G_3 - V_1 G_1 + \underline{E_1 G_1} + V_2 G_2 - V_1\cdot G_2 + \underline{E_2 G_2} - V_1 G_5 + V_4\cdot G_5 = 0 \\ V_4\cdot G_4 - V_2 G_4 + \underline{E_4 G_4} - V_2 G_2 + V_1\cdot G_2 - \underline{E_2 G_2} - V_2 G_6 + V_3 G_6 = 0 \\ V_2\cdot G_6 - V_3 G_6 - V_3\cdot G_3 + V_4 G_3 + \underline{E_3 G_3} - V_3\cdot G_1 + V_1\cdot G_1 - \underline{E_1\cdot G_1} = 0 \end{cases}$$

Assumption: $\underline{V_4 = 0}$

Counteractions

$$E_1\cdot G_1 + E_2\cdot G_2 = V_1(G_1+G_2+G_5) - \overbrace{V_3 G_1 - V_2\cdot G_2}^{\text{Counteractions}} - \underbrace{V_4 G_5}_{=0 \text{ because } V_4=0}$$

$$E_4 G_4 - E_2 G_2 = V_2(G_2+G_4+G_6) - V_1\cdot G_2 - V_3\cdot G_6 - \underbrace{V_4\cdot G_4}_{=0 \text{ because } V_4=0}$$

$$E_3\cdot G_3 - E_1\cdot G_1 = V_3(G_1+G_2+G_6) - V_1 G_1 - V_2\cdot G_6 - \underbrace{V_4\cdot G_4}_{=0 \text{ because } V_4=0}$$

Node's own conductances; Conductances of nodes which are in counteraction

$$\left(\sum J_s\right)_{I} = E_1\cdot G_1 + E_2\cdot G_2 = V_1\overbrace{(G_1+G_2+G_5)}^{\text{Node's own conductances}} - \overbrace{V_2\cdot G_2 - V_3\cdot G_1}^{\text{Conductances of nodes which are in counteraction}} - \underbrace{V_4\cdot G_5}_{=0,\ \to V_4=0}$$

$$\left(\sum J_s\right)_{II} = E_4 G_4 - E_2\cdot G_2 = V_2(G_2+G_4+G_6) - V_1\cdot G_2 - V_3 G_6 - \underbrace{V_4\cdot G_4}_{=0,\ \to V_4=0}$$

$$\left(\sum J_s\right)_{III} = E_3\cdot G_3 - E_1\cdot G_1 = V_3(G_1+G_3+G_6) - V_1\cdot G_1 - V_2\cdot G_6 - \underbrace{V_4\cdot G_3}_{=0,\ \to V_4=0}$$