http://www.mbstudent.com/electrical-engineering.html

Application of node voltage method to solve electric circuit.

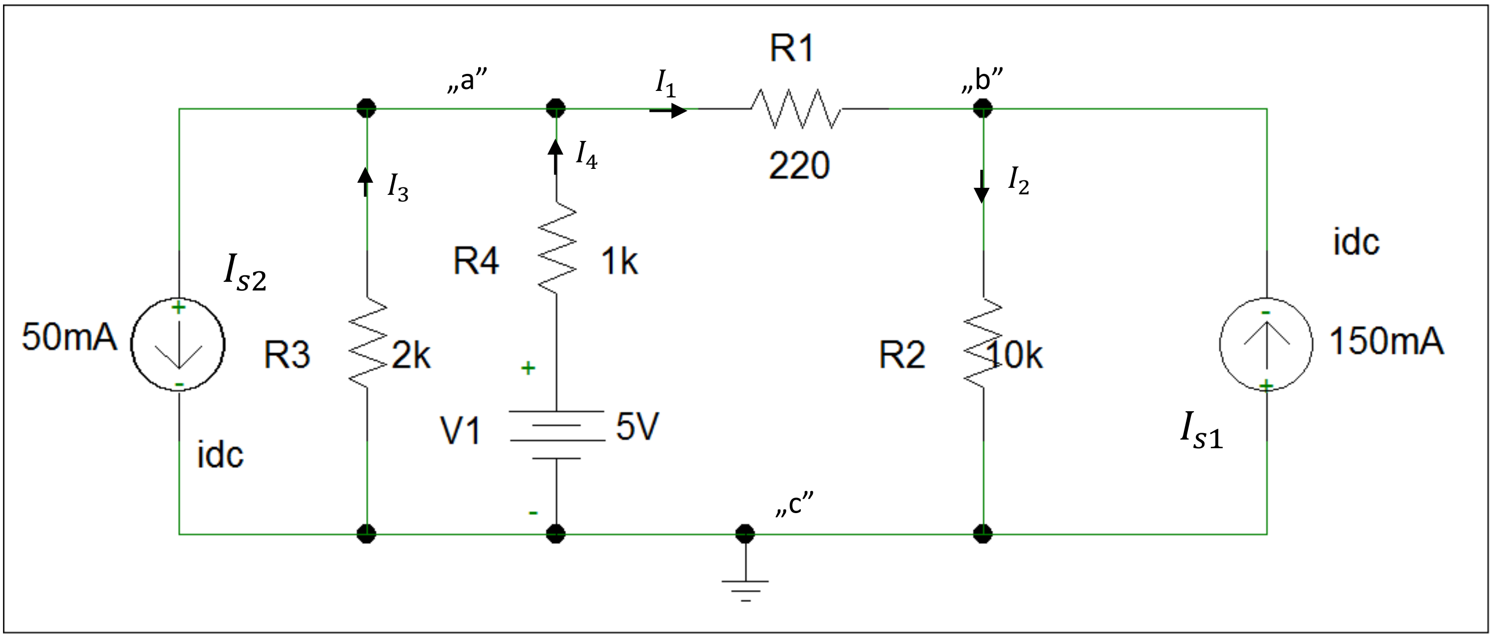
Node voltage method will be used to find expressions for currents and voltages in circuit. Node voltage method is based on Kirchhoff's current law (KCL). Number of equations for Kirchhoff's current law (KCL) and node voltage method is given by formula:

$$KCL \rightarrow n - 1$$

$$n - number\ of\ nodes$$

In circuit below are three nodes. Number of equations is then

$$KCL \rightarrow n - 1 = 3 - 1 = 2$$



Picture 1. Electric circuit to designate its currents and voltages.

In node voltage method we have to assume that one of nodes has potential $0[V]$. We connect to chosen node a ground. In this example we assume that node "c" has potential $0[V]$.

Equation for node "a"

$$\Sigma(I_s)_a = -I_{s2} + \frac{V1}{R4} = V_a \cdot (G3 + G4) - V_b \cdot G1 - V_c \cdot (G3 + G4)$$

Because we have assumed that potential of node "c" is zero → $V_c = 0[V]$, then equation for current sources in node "a" simplifies.

$$\Sigma(I_s)_a = -I_{s2} + \frac{V1}{R4} = V_a \cdot (G3 + G4) - V_b \cdot G1$$

Equation for node "b"

$$\Sigma(I_s)_b = I_{s1} = V_b \cdot (G1 + G2) - V_a \cdot G1 - V_c \cdot G2$$

Because we have assumed that potential of node "c" is zero → $V_c = 0[V]$, then equation for current sources in node "b" simplifies.

$$\Sigma(I_s)_b = I_{s1} = V_b \cdot (G1 + G2) - V_a \cdot G1$$

To designate values of potentials V_a and V_b we have to solve equation system. Expression for potential V_a we will receive from equation current sources sum in node "a".

$$-I_{s2} + \frac{V1}{R4} = V_a \cdot (G3 + G4) - V_b \cdot G1$$

$$V_a \cdot (G3 + G4) = V_b \cdot G1 - I_{s2} + V1 \cdot G4$$

$$V_a = V_b \cdot \frac{G1}{G3 + G4} - \frac{I_{s2}}{G3 + G4} + V1 \cdot \frac{G4}{G3 + G4}$$

Now we will insert expression for V_a to equation for current sources sum in node "b". As a result we will receive expression for potential V_b.

$$I_{s1} = V_b \cdot (G1 + G2) - V_a \cdot G1$$

$$V_b \cdot (G1 + G2) = I_{s1} + V_a \cdot G1$$

$$V_b = \frac{I_{s1}}{G1 + G2} + V_a \cdot \frac{G1}{G1 + G2}$$

$$V_b = \frac{I_{s1}}{G1 + G2} + \left(V_b \cdot \frac{G1}{G3 + G4} - \frac{I_{s2}}{G3 + G4} + V1 \cdot \frac{G4}{G3 + G4}\right) \cdot \frac{G1}{G1 + G2}$$

$$V_b = \frac{I_{s1}}{G1 + G2} - \frac{I_{s2}}{G3 + G4} \cdot \frac{G1}{G1 + G2} + V1 \cdot \frac{G4}{G3 + G4} \cdot \frac{G1}{G1 + G2}$$

$$V_b - V_b \cdot \frac{G1}{G3 + G4} \cdot \frac{G1}{G1 + G2} = \frac{I_{s1}}{G1 + G2} - \frac{I_{s2}}{G3 + G4} \cdot \frac{G1}{G1 + G2} + V1 \cdot \frac{G4}{G3 + G4} \cdot \frac{G1}{G1 + G2}$$

$$V_b \cdot \left(1 - \frac{G1}{G3 + G4} \cdot \frac{G1}{G1 + G2}\right) = \frac{I_{s1}}{G1 + G2} - \frac{I_{s2}}{G3 + G4} \cdot \frac{G1}{G1 + G2} + V1 \cdot \frac{G4}{G3 + G4} \cdot \frac{G1}{G1 + G2}$$

$$V_b = \frac{\frac{I_{s1}}{G1+G2} - \frac{I_{s2}}{G3+G4}\cdot\frac{G1}{G1+G2} + V1\cdot\frac{G4}{G3+G4}\cdot\frac{G1}{G1+G2}}{1-\frac{G1}{G3+G4}\cdot\frac{G1}{G1+G2}}$$

$$V_b = \frac{\frac{I_{s1}}{G1+G2} - \frac{I_{s2}}{G3+G4}\cdot\frac{G1}{G1+G2} + V1\cdot\frac{G4}{G3+G4}\cdot\frac{G1}{G1+G2}}{1-\frac{G1}{G3+G4}\cdot\frac{G1}{G1+G2}}$$

$$V_b = \frac{\frac{I_{s1}\cdot(G3+G4)}{(G1+G2)\cdot(G3+G4)} - \frac{I_{s2}\cdot G1}{(G1+G2)\cdot(G3+G4)} + V1\cdot\frac{G1\cdot G4}{(G1+G2)\cdot(G3+G4)}}{\frac{(G1+G2)\cdot(G3+G4)}{(G1+G2)\cdot(G3+G4)} - \frac{G1^2}{(G1+G2)\cdot(G3+G4)}}$$

All expression in equation for V_b are divided by the same factor. Equation will be simplified by eliminating this factor.

$$V_b = \frac{I_{s1}\cdot(G3+G4) - I_{s2}\cdot G1 + V1\cdot(G1\cdot G4)}{(G1+G2)\cdot(G3+G4) - G1^2}$$

Potential V_b is calculated. We will insert expression for V_b to equation for V_a.

$$V_a = V_b\cdot\frac{G1}{G3+G4} - \frac{I_{s2}}{G3+G4} + V1\cdot\frac{G4}{G3+G4}$$

$$V_a = \left(\frac{I_{s1}\cdot(G3+G4) - I_{s2}\cdot G1 + V1\cdot(G1\cdot G4)}{(G1+G2)\cdot(G3+G4) - G1^2}\right)\cdot\frac{G1}{G3+G4} - \frac{I_{s2}}{G3+G4} + V1\cdot\frac{G4}{G3+G4}$$

Expressions for potentials are known so we can calculate currents values.

$$I_1 = (V_a - V_b)\cdot G1$$

$$I_2 = (V_b - V_c)\cdot G2 = V_b\cdot G2$$

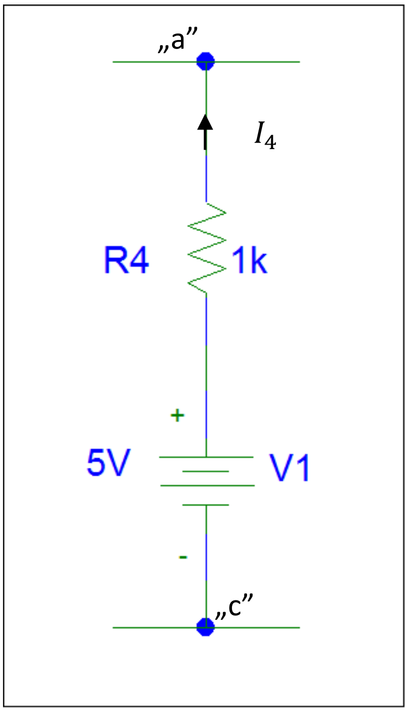
$$I_3 = (V_a - V_c)\cdot G3 = V_a\cdot G3$$

To calculate current I_4 we have to return from transformation to virtual current source to physical voltage source.

$$I_4 = \frac{V1}{R4} + (V_a - V_c)\cdot G4 = (V1 + V_a)\cdot G4$$

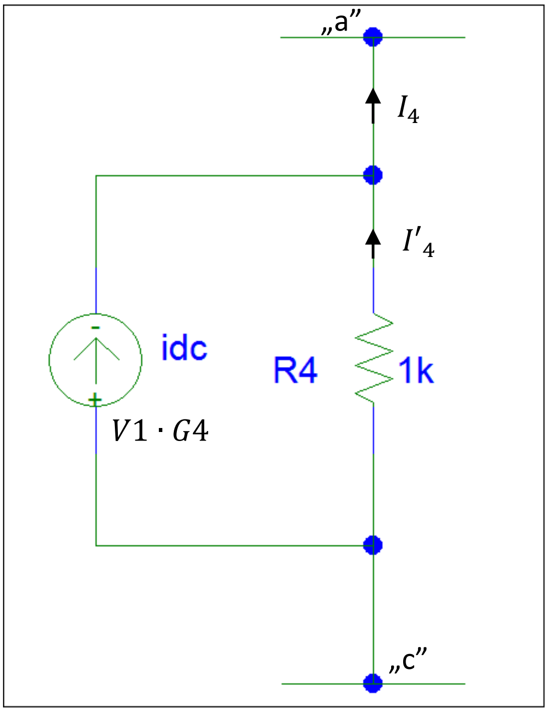
Explanation of expression for current I_4 is on the next page.

Because we applied node voltage method we had to transform all physical voltage sources to virtual current sources. Node voltage method has a rule that it does not see physical voltage source. Node voltage method sees only current sources.



Picture 2. Branch with resistor R4 and voltage source before transformation.

≡



Picture 3. Branch with resistor R4 and virtual current source after transformation.

As is shown on pictures above branch between nodes "a" and "c" have changed a little after its transformation. In all node voltage method calculation we used virtual current source from picture 3. Current I_4 is given by Kirchhoff's current (KCL) law.

$$V1 \cdot G4 + I'_4 - I_4 = 0$$

$$I_4 = V1 \cdot G4 + I'_4$$