http://www.mbstudent.com/electrical-engineering.html

Application of node voltage method to solve AC circuit.

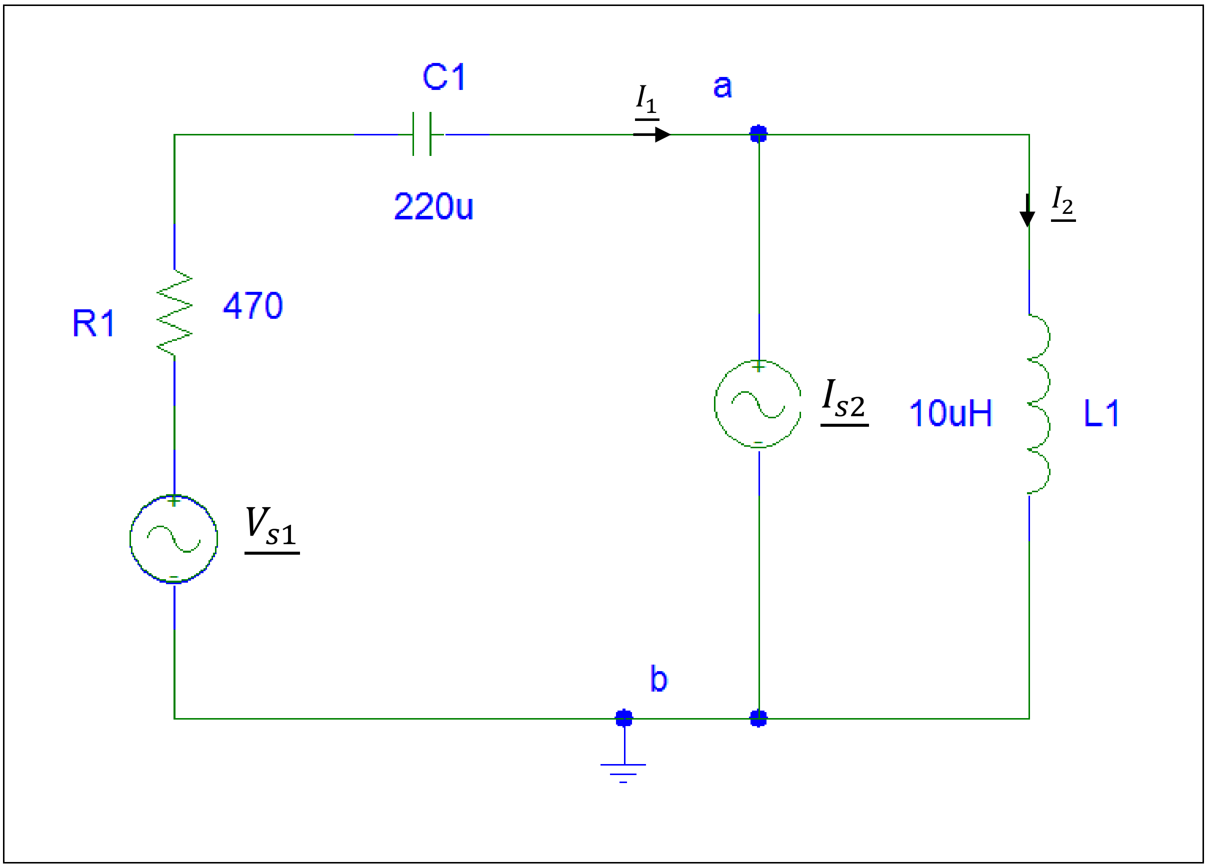
Node voltage method will be used to find expressions for currents and voltages in circuit. Node voltage method is based on Kirchhoff's current law (KCL). Number of equations for Kirchhoff's current law (KCL) and node voltage method is given by formula:

$$KCL \rightarrow n-1$$

$$n - number\ of\ nodes$$

In circuit below are two nodes. Number of equations is then

$$KCL \rightarrow n-1 = 2-1 = 1$$



Picture 1. Electric AC circuit.

In node voltage method we have to assume that one of nodes has potential $0[V]$. We connect to chosen node a ground. In this example we assume that node "b" has potential $0[V]$. We use node voltage method for analysis AC circuit. We have to remember that impedance's vector has three components: resistance R, inductive reactance X_L and capacitive reactance X_C. Considered circuit is AC circuit so currents and voltages are described by complex numbers.

Remember also about relation between impedance and admittance.

$$\underline{Y} = \frac{1}{\underline{Z}} \; and \; \underline{Z} = \frac{1}{\underline{Y}}$$

Equation for node "a"

$$\Sigma\left(\underline{I_s}\right)_a = \underline{V_{s1}} \cdot \underline{Y_{R1C1}} + \underline{I_{s2}} = \underline{V_a} \cdot \left(\underline{Y_{R1C1}} + \underline{Y_{L1}}\right) - \underline{V_b} \cdot \left(\underline{Y_{R1C1}} + \underline{Y_{L1}}\right)$$

$$\underline{Y_{R1C1}} = \frac{1}{R1 - j\frac{1}{\omega \cdot C}}$$

$$\underline{Y_{L1}} = \frac{1}{j \cdot \omega \cdot L}$$

Because we have assumed that potential of node "b" is zero $\underline{V_b} = 0$. We are allowed to simplify equation for current sources sum in node "a".

$$\Sigma\left(\underline{I_s}\right)_a = \underline{V_{s1}} \cdot \underline{Y_{R1C1}} + \underline{I_{s2}} = \underline{V_a} \cdot \left(\underline{Y_{R1C1}} + \underline{Y_{L1}}\right)$$

$$\underline{V_{s1}} \cdot \underline{Y_{R1C1}} + \underline{I_{s2}} = \underline{V_a} \cdot \left(\underline{Y_{R1C1}} + \underline{Y_{L1}}\right)$$

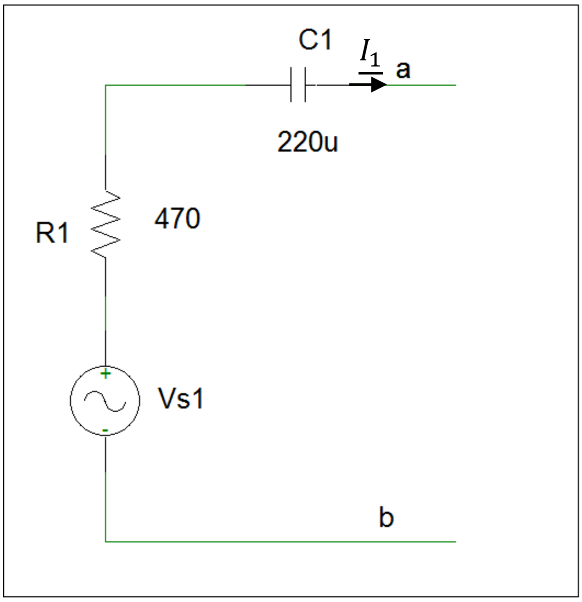
$$\underline{V_a} = \frac{\underline{\underline{V_{s1}} \cdot \underline{Y_{R1C1}} + \underline{I_{s2}}}}{\left(\underline{Y_{R1C1}} + \underline{Y_{L1}}\right)}$$

Now we can calculate currents in circuit. We have to take under consideration fact that one of current source was virtual. To calculate current $\underline{I_1}$ we have return from transformation to virtual current source.

$$\underline{I_1} = \underline{V_{s1}} \cdot \underline{Y_{R1C1}} + \underline{V_a} \cdot \underline{Y_{R1C1}}$$

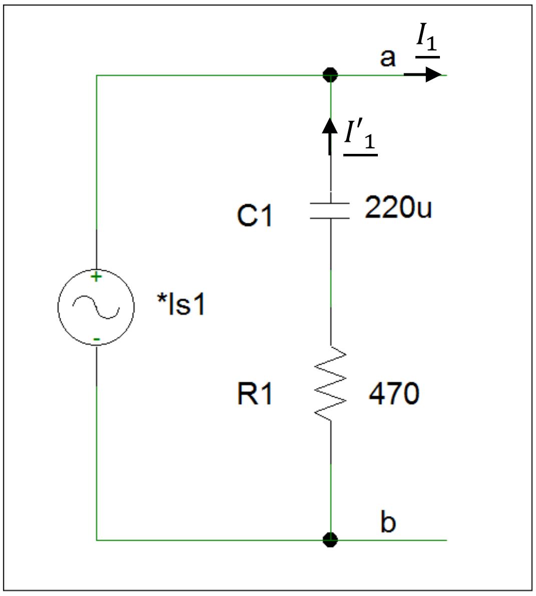
$$\underline{I_2} = \underline{V_a} \cdot \underline{Y_{L1}}$$

Explanation of equation for current $\underline{I_1}$ is on the next page.



Picture 2. **Branch with resistor $R1$ capacitor $C1$ and voltage source $\underline{V_{s1}}$ before transformation.**

≡



Picture 3. Branch with resistor $R1$ capacitor $C1$ and virtual current source $\underline{*I_{s1}}$ after transformation.

As is shown on pictures above branch between nodes "a" and "b" have changed a little after its transformation. In all node voltage method calculation we used virtual current source from picture 3. Current $\underline{I_1}$ is given by Kirchhoff's current (KCL) law.

$$\underline{*I_{s1}} + \underline{I'_1} - \underline{I_1} = 0$$

$$\underline{I_1} = \underline{*I_{s1}} + \underline{I'_1}$$

$$\underline{I_1} = \underline{V_{s1}} \cdot \underline{Y_{R1C1}} + (\underline{V_a} - \underline{V_b}) \cdot \underline{Y_{R1C1}}$$

Because we have assumed that potential $\underline{V_b} = 0$

$$\underline{I_1} = \underline{V_{s1}} \cdot \underline{Y_{R1C1}} + \underline{V_a} \cdot \underline{Y_{R1C1}}$$

$$\underline{I_1} = \underline{V_{s1}} \cdot \frac{1}{R1 - j\frac{1}{\omega \cdot C}} + \underline{V_a} \cdot \frac{1}{R1 - j\frac{1}{\omega \cdot C}}$$