$$y'' + y = \begin{cases} 0 & \text{dla } t < 1 \\ 1 & \text{dla } 1 \leq t \leq 3 \\ 0 & \text{dla } t > 0 \end{cases} \qquad y(0) = 0 \quad y'(0) = 0$$

$$\eta(t) = 1(t)$$

$$y'' + y = \eta(t-1) - \eta(t-3)$$

$$\mathcal{L}\{y\} = Y(s) \quad ; \quad \mathcal{L}\{y''\} = s^2 \cdot Y(s) + \underbrace{s \cdot y(0) - \overset{0}{y'(0)}}_{=0} \quad ; \quad \mathcal{L}\{\eta(t-1)\} = \frac{1}{s} \cdot e^{-s}$$

$$\mathcal{L}\{\eta(t-3)\} = \frac{1}{s} \cdot e^{-3 \cdot s}$$

$$\mathcal{L}\{f(t-a)\} = F(s) \cdot e^{-a \cdot s}$$

$$\mathcal{L}[e^{-a \cdot t} \cdot f(t)] = F(s-a)$$

Equation write after transformation:

$$s^2 \cdot Y(s) + Y(s) = \frac{1}{s} - \frac{1}{s} \cdot e^{-3 \cdot s}$$

$$Y(s) \cdot (s^2+1) = \frac{1}{s} - \frac{1}{s} \cdot e^{-3 \cdot s}$$

$$Y(s) = \frac{1}{s(s^2+1)} \cdot e^{-s} \quad \frac{1}{s(s^2+1)} \cdot e^{-3 \cdot s}$$

$$Y(s) = \frac{1}{s \cdot (s^2+1))} \cdot e^{-s} - \frac{1}{s \cdot (s^2+1))} \cdot e^{-3 \cdot s}$$

$$\frac{1}{s(s^2+1)} = \frac{A}{s} + \frac{B \cdot s + C}{s^2+1}$$

$$1 = \frac{\cancel{s}(s^2+1) \cdot A}{\cancel{s}} + \frac{s\cancel{(s^2+1)} \cdot (B \cdot s + C)}{\cancel{(s^2+1)}} = s^2 \cdot A + A + s^2 \cdot B + s \cdot C$$

$$\begin{array}{c|l} s^2 & A + B = 0 \rightarrow B = -1 \quad B = -1 \\ s^1 & C = 0 \\ s^0 & A = 1 \end{array}$$

$$Y(s) = \left(\frac{1}{s} - \frac{s}{s^2+1}\right) \cdot e^{-s} - \left(\frac{1}{s} - \frac{s}{s^2+1}\right) \cdot e^{-3 \cdot s}$$

$$y(t) = (1 - \cos t)\Big|_{t := t-1} \cdot \eta(t-1) - (1 - \cos t)\Big|_{t := t-3} \cdot \eta(t-3)$$

$$y(t) = (1 - \cos(t-1)) \cdot \eta(t-1) - (1 - \cos(t-3)) \cdot \eta(t-3)$$