$$y''+2y'+5y=\begin{cases}0 & t<0\\ 1 & 0<t\le 2\\ 0 & t>2\end{cases} \qquad y(0)=1 \quad y'(0)=0$$

$\mathcal{L}\{y\}=Y(s)$; $\mathcal{L}\{y'\}=s\cdot Y(s)+s^0\cdot y(0)=s\cdot Y(s)-1$; $\mathcal{L}\{y''\}=s^2\cdot Y(s)-s\cdot y(0)-s^0\cdot y'(0)$

$\mathcal{L}\{1(t)\}=\frac{1}{s}$
$\mathcal{L}\{t^n\}=\frac{n!}{s^{n+1}}$
$\mathcal{L}\{\sin kt\}=\frac{k}{s^2+k^2}$
$\mathcal{L}\{\cos kt\}=\frac{s}{s^2+k^2}$
$\mathcal{L}\{f(t-a)\}=F(s)\cdot e^{-a\cdot s}$
$\mathcal{L}\{e^{a\cdot t}f(t)\}=F(s-a)$

$$s^2\cdot Y(s)-s+2\cdot(s\cdot Y(s)-1)+5\cdot Y(s)=\frac{1}{s}-\frac{1}{s}\cdot e^{-2\cdot s}$$
$$s^2\cdot Y(s)-s+2\cdot s\cdot Y(s)-2+5\cdot Y(s)=\frac{1}{s}-\frac{1}{s}\cdot e^{-2\cdot s}$$
$$s^2\cdot Y(s)+2\cdot Y(s)+5Y(s)=\frac{1}{s}-\frac{1}{s}\cdot e^{-2\cdot s}+2+s$$
$$Y(s)\cdot(s^2+2\cdot s+5)=\frac{1}{s}-\frac{1}{s}\cdot e^{-2\cdot s}+2+s$$
$$Y(s)=\frac{\frac{1}{s}-\frac{1}{s}\cdot e^{-2\cdot s}+2+s}{(s^2+2\cdot s+5)}$$

$\Delta=b^2-4\cdot a\cdot c=2^2-4\cdot 1\cdot 5=-16\rightarrow \Delta<0$

$$Y(s)=\frac{1}{s(s^2+2s+5)}-\frac{1}{s(s^2+2\cdot s+5)}\cdot e^{-2\cdot s}+\frac{2+s}{s^2+2s+5}$$

$$\frac{1}{s(s^2+2s+5)}=\frac{A}{s}+\frac{B\cdot s+C}{s^2+2s+5}$$

$$1=A\cdot(s^2+2\cdot s+5)+s(B\cdot s+C)$$
$$1=\underline{A\cdot s^2}+\underline{2\cdot A\cdot s}+5A+\underline{B\cdot s^2}+\underline{C\cdot s}$$

s^2	$A+B=0 \rightarrow B=-A=-\frac{1}{5}$
s^1	$2A+C=0\rightarrow C=-2\cdot A=-\frac{2}{5}$
s^0	$5A=1\rightarrow A=\frac{1}{5}$

$A=\frac{1}{5}$; $B=-\frac{1}{5}$; $C=-\frac{2}{5}$

$$Y(s)=\frac{s+2}{s^2+2s+5}+\frac{1}{5}\cdot\frac{1}{s}+\frac{-\frac{1}{5}\cdot s-\frac{2}{5}}{s^2+2\cdot s+5}-\left[\frac{1}{5}\cdot\frac{1}{s}+\frac{-\frac{1}{5}\cdot s-\frac{2}{5}}{s^2+2\cdot s+5}\right]\cdot e^{-2\cdot s}$$

$s^2+2\cdot s+5=(s+1)^2+4$

$$Y(s)=\frac{s}{(s+1)^2+4}+\frac{2}{(s+1)^2+4}+\frac{1}{5}\cdot\frac{1}{s}-\frac{1}{5}\cdot\frac{s}{(s+1)^2+4}-\frac{2}{5}\cdot\frac{1}{(s+1)^2+4}+$$
$$-\left[\frac{1}{5}\cdot\frac{1}{s}-\frac{1}{5}\cdot\frac{s}{(s+1)^2+4}-\frac{2}{5}\cdot\frac{1}{(s+1)^2+4}\right]\cdot e^{-2\cdot s}$$

$$Y(s)=\left.\frac{s}{s^2+4}\right|_{s:=s+1}+\left.\frac{2}{2}\frac{2}{s^2+4}\right|_{s:=s+1}+\frac{1}{5}\cdot\frac{1}{s}-\left.\frac{1}{5}\cdot\frac{s}{s^2+4}\right|_{s:=s+1}-\frac{1}{5}\cdot\frac{1}{s}\cdot e^{-2\cdot s}+\left.\frac{1}{5}\cdot\frac{s}{s^2+1}\right|_{s:=s+1}\cdot e^{-2\cdot s}$$
$$+\left.\frac{4}{10}\cdot\frac{1}{s^2+4}\right|_{s:=s+1}\cdot e^{-2\cdot s}$$

↳ 3

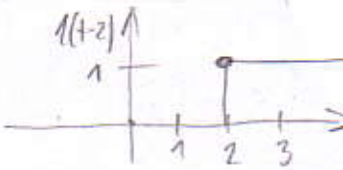
$$y(t) = e^{t}\cdot\cos 2t + \frac{1}{2}\cdot e^{t}\cdot\sin 2t + \frac{1}{5} - \frac{1}{5}\cdot e^{t}\cdot\cos 2t - \frac{1}{5}\cdot 1(t)\Big|_{t:=t-2}\cdot e^{-t}$$

$$+ \frac{1}{5}\cdot e^{t}\cdot\cos t\Big|_{t:=t-2} + \frac{1}{10}\cdot e^{t}\cdot\sin t\Big|_{t:=t-2}$$

$$y(t) = e^{-t}\cdot\cos 2t + \frac{1}{2}\cdot e^{-t}\cdot\sin 2t + \frac{1}{5} - \frac{1}{5}\cdot e^{-t}\cdot\cos 2t + \left[-\frac{1}{5}\cdot e^{-(t-2)}\cdot 1(t-2) + \right.$$

$$\left. + \frac{1}{5}\cdot e^{-(t-2)}\cos(t-2) + \frac{1}{10}\cdot e^{-(t-2)}\cdot\sin(t-2)\right]\cdot 1(t-2)$$

$1(t-2) = 1(t-2)$



$1(t-2) = \begin{cases} 1 & \text{dla } t \geq 2 \\ 0 & \text{dla } t < 2 \end{cases}$

$$\mathcal{L}\{e^{a\cdot t}\cdot f(t)\} = F(s-a) \qquad \mathcal{L}\{f(t-a)\} = F(s)\cdot e^{-a\cdot s}$$

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$$y'' + 8\cdot y' + 16\cdot y = \begin{cases} 0 & t<2 \\ 1 & 2\leq t\leq 4 \\ 0 & t>4 \end{cases} \qquad y(0)=1 \quad y'(0)=0$$

$\mathcal{L}\{e^{a\cdot t}\cdot f(t)\} = F(s-a)$

$\mathcal{L}\{f(t-a)\} = F(s)\cdot e^{-a\cdot s}$

$\mathcal{L}\{y(t)\} = Y(s)$; $\mathcal{L}\{y'(t)\} = s\cdot Y(s) - s^{0}\cdot y(0) = s\cdot Y(s) - 1$;

$\mathcal{L}\{y''(t)\} = s^{2}\cdot Y(s) - s\cdot y(0) - s^{0}\cdot y'(0) = s^{2}\cdot Y(s) - s - 0 = s^{2}\cdot Y(s) - s$

$$s^{2}\cdot Y(s) - s + 8\cdot(s\cdot Y(s) - 1) + 16\cdot Y(s) = \frac{1}{5}\cdot e^{-2\cdot s} - \frac{1}{5}\cdot e^{-4\cdot s}$$

$$s^{2}\cdot Y(s) - s + 8\cdot s\cdot Y(s) - 8 + 16\cdot Y(s) = \frac{1}{5}\cdot e^{-2\cdot s} - \frac{1}{5}\cdot e^{-4\cdot s}$$

$$s^{2}\cdot Y(s) + 8\cdot s\cdot Y(s) + 16\cdot Y(s) = \frac{1}{5}\cdot e^{-2\cdot s} - \frac{1}{5}\cdot e^{-4\cdot s} + s + 8$$

$$Y(s)\cdot(s^{2} + 8\cdot s + 16) = \frac{1}{5}\cdot e^{-2\cdot s} - \frac{1}{5}\cdot e^{-4\cdot s} + s + 8$$

$$Y(s)\cdot(s+4)^{2} = \frac{1}{5}\cdot e^{-2\cdot s} - \frac{1}{5}\cdot e^{-4\cdot s} + s + 8$$

$$Y(s) = \frac{1}{5\cdot(s+4)^{2}}\cdot e^{-2\cdot s} - \frac{1}{5\cdot(s+4)^{2}}\cdot e^{-4\cdot s} + \frac{s+8}{(s+4)^{2}}$$

$$\frac{1}{s\cdot(s+4)^{2}} = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{(s+4)^{2}}$$

$$1 = A\cdot(s+4)^{2} + B\cdot s\cdot(s+4) + C\cdot s$$

$$1 = A\cdot s^{2} + A\cdot 8\cdot s + A\cdot 16 + B\cdot s^{2} + B\cdot 4\cdot s + C\cdot s$$

$A = \frac{1}{16}$, $B = -\frac{1}{16}$, $C = -\frac{1}{4}$

s^{2}	$A+B=0 \rightarrow B=-A=-\frac{1}{16}$
s^{1}	$8A+4B+C=0$
s^{0}	$16\cdot A=1 \rightarrow A=\frac{1}{16}$

$C = -8A + 4\cdot B$

$C = -\frac{8}{16} - \left(-\frac{4}{16}\right)$

$C = -\frac{4}{16} = -\frac{1}{4}$

$$Y(s)=\left(\frac{1}{16}\cdot\frac{1}{s}-\frac{1}{16}\cdot\frac{1}{s+4}-\frac{1}{4}\cdot\frac{1}{(s+4)^2}\right)\cdot e^{-2\cdot s}-\left(\frac{1}{16}\cdot\frac{1}{s}-\frac{1}{16}\cdot\frac{1}{s+4}-\frac{1}{4}\cdot\frac{1}{(s+4)^2}\right)\cdot e^{-4\cdot s}+$$

$$+\frac{s}{(s+4)^2}+\frac{8}{(s+4)^2}$$

$$Y(s)=\left(\frac{1}{16}\cdot\frac{1}{s}-\frac{1}{16}\cdot\frac{1}{s+4}-\frac{1}{4}\cdot\frac{1}{(s+4)^2}\right)\cdot e^{-2\cdot s}-\left(\frac{1}{16}\cdot\frac{1}{s}-\frac{1}{16}\cdot\frac{1}{s+4}-\frac{1}{4}\cdot\frac{1}{(s+4)^2}\right)\cdot e^{-4\cdot s}+$$

$$+\frac{s+4-4}{(s+4)^2}+\frac{8}{(s+4)^2}$$

$$Y(s)=\left(\frac{1}{16}\cdot\frac{1}{s}-\frac{1}{16}\cdot\frac{1}{s+4}-\frac{1}{4}\cdot\frac{1}{(s+4)^2}\right)\cdot e^{-2\cdot s}-\left(\frac{1}{16}\cdot\frac{1}{s}-\frac{1}{16}\cdot\frac{1}{s+4}-\frac{1}{4}\cdot\frac{1}{(s+4)^2}\right)\cdot e^{-4\cdot s}+$$

$$+\frac{\cancel{s+4}}{(s+4)^2}+\frac{4}{(s+4)^2}\quad\hookrightarrow\frac{2}{s}=\frac{8}{(s+4)^2}$$

$$y(t)=\left[\frac{1}{16}\cdot 1(t)\Big|_{t:=t-2}-\frac{1}{16}\cdot e^{-4\cdot t}\Big|_{t:=t-2}-\frac{1}{4}\cdot e^{-4\cdot t}\cdot t\Big|_{t:=t-2}\right]-\left[\frac{1}{16}\cdot 1(t)\Big|_{t:=t-4}-\frac{1}{16}\cdot e^{-4\cdot t}\Big|_{t:=t-4}-\frac{1}{4}\cdot e^{-4\cdot t}\cdot t\Big|_{t:=t-4}\right]$$

$$+e^{-4\cdot t}+4\cdot e^{-4\cdot t}\cdot t$$

$$y(t)=\left[\frac{1}{16}\cdot 1(t-2)-\frac{1}{16}\cdot e^{-4(t-2)}-\frac{1}{4}\cdot e^{-4(t-2)}\cdot(t-2)\right]\cdot\eta(t-2)-\left[\frac{1}{16}\cdot 1(t-4)-\frac{1}{16}\cdot e^{-4\cdot(t-4)}-\frac{1}{4}\cdot e^{-4(t-4)}\cdot(t-4)\right]\eta$$

$$+e^{-4\cdot t}+4\cdot e^{-4\cdot t}\cdot t$$