http://www.mbstudent.com/maths-examples.html

Laplace transformation example

Solve differential equation with application of Laplace transformation

$$y(0) = 0; y'(0) = 0$$

$$y + y' = -3$$

$$y + y' + 3 = 0$$

$$Y(s) + s \cdot Y(s) + \frac{3}{s} = 0$$

$$Y(s) + s \cdot Y(s) = -\frac{3}{s}$$

$$Y(s) \cdot (1 + s) = -\frac{3}{s}$$

$$Y(s) = -\frac{3}{s \cdot (1 + s)}$$

$$\frac{-3}{s \cdot (1 + s)} = \frac{A}{s} + \frac{B}{1 + s}$$

$$A \cdot (1 + s) + B \cdot s$$

$$-3 = A \cdot s + A + B \cdot s$$

$$\begin{array}{l|c} s^1 & A + B = 0 \rightarrow B = -A \rightarrow B = 3 \\ s^0 & A = -3 \end{array}$$

$$Y(s) = \frac{A}{s} + \frac{B}{1 + s}$$

$$y(t) = A + B \cdot e^{-t}$$

$$y(t) = -3 + 3 \cdot e^{-t}$$