

Definite integral example

Calculate definite integral of function for border values

$$\int_0^1 x \cdot e^{2x} \cdot dx$$

We will use integration by parts method.

$$\int u \cdot dv = u \cdot v - \int v \cdot du$$

$$\int_0^1 x \cdot e^{2x} \cdot dx = \begin{bmatrix} u = x & dv = e^{2x} \\ du = 1 & v = \frac{1}{2} \cdot e^{2x} \end{bmatrix}$$

$$\left[x \cdot \frac{1}{2} \cdot e^{2x} \right]_0^1 - \int_0^1 \frac{1}{2} \cdot e^{2x} \cdot dx$$

At this point to calculate equation we will use substitution method. We have to remember that in substitution method values of border values also are changing.

$$\left[x \cdot \frac{1}{2} \cdot e^{2x} \right]_0^1 - \int_0^1 \frac{1}{2} \cdot e^{2x} \cdot dx$$

$$\int_0^1 \frac{1}{2} \cdot e^{2x} \cdot dx = \begin{bmatrix} 2 \cdot x = t \\ 2 \cdot dx = dt \\ dx = \frac{dt}{2} \end{bmatrix} = \int_0^2 \frac{1}{2} \cdot e^t \cdot \frac{dt}{2} = \frac{1}{4} \cdot \int_0^2 e^t \cdot dt$$

$$\frac{1}{4} \cdot \int_0^2 e^t \cdot dt = \frac{1}{4} \cdot [e^t]_0^2 = \frac{1}{4} \cdot [e^2 - e^0]$$

$$\left[x \cdot \frac{1}{2} \cdot e^{2x} \right]_0^1 - \left(\frac{1}{4} \cdot [e^t]_0^2 \right)$$

$$\left[1 \cdot \frac{1}{2} \cdot e^{2 \cdot 1} - 0 \cdot \frac{1}{2} \cdot e^{2 \cdot 0} \right] - \left(\frac{1}{4} \cdot [e^2 - e^0] \right)$$

$$\frac{1}{2} \cdot e^2 - \frac{1}{4} \cdot e^2 + \frac{1}{4} \cdot e^0$$

$$\frac{1}{4} \cdot (e^2 + 1)$$