

$$x \cdot y' + y = 2x \quad / : x$$

$$y' + \frac{y}{x} = 2$$

$$GIE = SIE + GHE$$

Homogeneous equation:

$$\frac{dy}{dx} + \frac{y}{x} = 0$$

$$\frac{dy}{dx} = -\frac{y}{x} = 0$$

$$-\int \frac{dy}{y} = \int \frac{dx}{x} \quad y=0$$

$$-\ln|y| + C_a = \ln|x| + C_b \quad \text{sub}$$

$$C_a + C_a = C^*$$

$$\ln C_0 = C^*$$

take constant
↓
 $|y| = -e^{-(\ln|x| + C^*)}$

$$|y| = e^{-\ln|x|} \cdot (e^C) = C$$

$$|y| = C \cdot e^{-\ln|x|}$$

$$\begin{cases} y = \pm C \cdot e^{-\ln|x|} \\ y = 0 \end{cases}$$

$$y' = \frac{\pm C \cdot e^{-\ln|x|}}{x}$$

$$\ln|y| = -\ln|x| + C^*$$

$$\ln|y| = \ln C_0^* - \ln|x|$$

$$\ln|y| = \ln \frac{C_0}{|x|}$$

$$|y| = \frac{C_0}{|x|}$$

$$y = \frac{(C_0)}{\pm x} \rightarrow C$$

$$y = \frac{C}{x}$$

General homogeneous equation:

$$\begin{cases} y = \frac{C}{x} \\ y = 0 \end{cases}$$

Variation of constant $C \rightarrow C(x)$:

$$y = \frac{C(x)}{x} = (x) \cdot x^{-1} \Rightarrow y' = \frac{C'(x)}{x} - \frac{C(x)}{x^2} = \frac{C'(x) \cdot x - C(x)}{x^2}$$

$$y' = \frac{C'(x)}{x} - \frac{C(x)}{x^2} = \frac{C'(x) \cdot x - C(x)}{x^2}$$

$$\frac{C'(x) \cdot x - C(x)}{x^2} + \frac{1}{x} \cdot \frac{C(x)}{x} = 2$$

$$\frac{C'(x) \cdot x}{x^2} - \frac{\cancel{C(x)}}{\cancel{x^2}} + \frac{\cancel{C(x)}}{\cancel{x^2}} = 2$$

$$\frac{C'(x)}{x} = 2 \Rightarrow C'(x) = 2x$$

$$C(x) = \int 2x \cdot dx = 2 \cdot \int x \cdot dx = 2 \cdot \frac{x^2}{2} + C_M$$

GIE after inserting C(x):

$$y = \frac{C(x)}{x} = \frac{x^2 + C_M}{x} = x + \frac{C_M}{x}$$

$$\boxed{y = x + \frac{C_M}{x}}$$

$$\text{GIE} = \text{SIE} + \text{GHE}$$

GIE - general inhomogeneous equation

SIE - specific inhomogeneous equation

GHE - general homogeneous equation