$$y' + 2 \cdot x \cdot y = x \qquad y(0) = 1$$

Homogeneous equation:

$$y' + 2 \cdot x \cdot y = 0 \qquad y(0) = 1$$

$$\frac{dy}{dx} + 2 \cdot x \cdot y = 0$$

$$\frac{dy}{dx} = -2 \cdot x y$$

$$\int \frac{dy}{y} = \int -2 \cdot x \cdot dx$$

$$\int \frac{dy}{y} = -2 \cdot \int x \cdot dx$$

$$\ln|y| + C_a = -\cancel{2} \cdot \frac{x^2}{\cancel{2}} + C_b$$

$$\ln|y| = -x^2 + C_a - C_b \qquad C_a - C_b = C_c$$

$$\ln|y| = -x^2 + C_c$$

$$|y| = e^{-x^2 + C_c}$$

$$y = \pm \cdot e^{-x^2 + C_c}$$

$$y = \pm \cdot e^{-x^2} \cdot e^{C_c} \qquad \left\{ \pm e^{C_c} = C \right.$$

$$y = C \cdot e^{-x^2} \rightarrow$$

Variation of constant: $y = C(x) \cdot e^{-x^2}$

$$y' = C'(x) \cdot e^{-x^2} + C(x) \cdot (-2) \cdot x \cdot e^{-x^2}$$

Inserting into inhomogeneous equation: $(y' + 2 \cdot x \cdot y = x)$

$$C'(x) \cdot e^{-x^2} \cancel{- 2 \cdot C(x) \cdot x \cdot e^{-x^2}} \cancel{+ 2 \cdot C(x) \cdot x \cdot e^{-x^2}} = x$$

$$C'(x) \cdot e^{-x^2} = x \quad / : e^{-x^2}$$

$$C'(x) = \frac{x}{e^{-x^2}} \rightarrow C'(x) = x \cdot e^{x^2}$$

$$\int C'(x) = \int x \cdot e^{x^2} \cdot dx = \left\{ \begin{array}{l} x^2 = t \\ 2x \cdot dx = dt \\ x \cdot dx = \frac{1}{2} \cdot dt \end{array} \right.$$

$$C(x) = \int \frac{1}{2} \cdot e^t \cdot dt$$

$$C(x) = \frac{1}{2} \cdot e^t + C_x$$

$$C(x) = \frac{1}{2} \cdot e^{x^2} + C_x$$

GIE=SIE+GHE:

$$y = C \cdot e^{-x^2} + \frac{1}{2} \cdot e^{x^2} \cdot e^{-x^2}$$

$$y = C \cdot e^{-x^2} + \frac{1}{2}$$

Condition: $y(0) = 1$

$$1 = C \cdot e^0 + \frac{1}{2}$$

$$C = 1 - \frac{1}{2} = \frac{1}{2}$$

$$y = \frac{1}{2} \cdot e^{-x^2} + \frac{1}{2}$$

$$\boxed{y = \frac{1}{2} \cdot (e^{-x^2} + 1)}$$

GIE=SIE+GHE

GIE - general inhomogeneous equation
SIE - specific inhomogeneous equation
GHE - general homogeneous equation