$y' + 2 \cdot x\, y = x$ ☺ $y(0) = 1$

GIE=SIE+GHE
GIE - general inhomogeneous equation
SIE - specific inhomogeneous equation
GHE - general homogeneous equation

General homogeneous equation:

$$y' + 2 \cdot x\, y = 0$$

$$\frac{dy}{dx} + 2 \cdot x \cdot y = 0$$

$$\frac{dy}{dx} = -2 \cdot x \cdot y$$

$$\int \frac{dy}{y} = \int 2 \cdot x \cdot dx$$

$$\ln|y| + C_a = -2 \cdot \frac{x^2}{2} + C_b$$

$$\ln|y| = -x^2 + \underbrace{(C_b - C_a)}_{= C_c}$$

$$\ln|y| = -x^2 + C_c$$

$$|y| = e^{-x^2 + C_c}$$

$$y = \pm \cdot e^{-x^2 + C_c}$$

$$y = \underbrace{\pm \cdot e^{C_c}}_{= C} \cdot e^{-x^2}$$

$$\underline{y = C \cdot e^{-x^2}}$$

Specific inhomogeneous equation:

$$y = C(x) \cdot e^{-x^2}$$

$$y' = C'(x) \cdot e^{-x^2} + C(x) \cdot e^{-x^2} \cdot (-2) \cdot x$$

After inserting to: ☺

$$C'(x) \cdot e^{-x^2} + \cancel{C(x) \cdot e^{-x^2} \cdot (-2) \cdot x} + \cancel{2 \cdot x \cdot C(x) \cdot e^{-x^2}} = x$$

$$C'(x) \cdot e^{-x^2} = x$$

$$C'(x) = x \cdot e^{x^2}$$

$$C(x) = \int x \cdot e^{x^2} \cdot dx = \left\{ \begin{array}{l} x^2 = t \\ 2x \cdot dx = dt \\ dx = \frac{dt}{2x} \end{array} \right\}$$

$$C(x) = \int \frac{1}{2} \cdot e^{t}\, dt = \frac{1}{2} \cdot e^{t} + C_D$$

$$C(x) = \frac{1}{2} \cdot e^{x^2} + C_D$$

GIE=SIE+GHE:

$$y = C \cdot e^{-x^2} + \frac{1}{2} \cdot e^{x^2} e^{-x^2}$$

$$y = C \cdot e^{-x^2} + \frac{1}{2}$$

take into account: $y(0) = 1$

$$1 = C \cdot e^{0} + \frac{1}{2}$$

$$C = 1 - \frac{1}{2} = \frac{1}{2}$$

$$y = \frac{1}{2} \cdot e^{-x^2} + \frac{1}{2}$$

$$\underline{y = \frac{1}{2} \cdot (e^{-x^2} + 1)}$$ → <u>GIE</u>=SIE+GHE:

http://www.mbstudent.com/maths-examples.html