Constant variation method:

$$\begin{cases} C_1'(x)\cdot y_1(x) + C_2'(x)\cdot y_2(x) = 0 \\ C_1'(x)\cdot y_1'(x) + C_2'(x)\cdot y_2'(x) = \frac{1}{a}\cdot g(x) \end{cases}$$

General inhomogeneous equation

$$y = y_c(x) + C_1\cdot y_1(x) + C_2\cdot y_2(x)$$

$y_c(x)$: Specific inhomogeneous equation

$C_1\cdot y_1(x) + C_2\cdot y_2(x)$: General homogeneous equation

$$W = \begin{vmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{vmatrix}$$

Example:

$$y'' + 4y = \frac{1}{\cos 2x}$$

pod.

$$y'' + 4y = 0 \rightarrow r^2 + 4 = 0 \quad r^2 = -4 \quad r = \sqrt{-4} \quad r = \pm 2i$$

$$y = C_1\cdot e^{0\cdot x}\cdot\cos 2x + C_2\cdot e^{0\cdot x}\cdot\sin 2x$$

Variation of constants:

$$y = C_1(x)\cdot\cos 2x + C_2(x)\cdot\sin 2x$$

$$y' = \underbrace{C_1'(x)\cdot\cos 2x}_{y_1} + C_1(x)\cdot\underbrace{(-2)\cdot\sin 2x}_{y_1'} + \underbrace{C_2'(x)\cdot\sin 2x}_{y_2} + C_2(x)\cdot\underbrace{2\cdot\cos 2x}_{y_2'}$$

$$\begin{cases} C_1'(x)\cdot\cos 2x + C_2'(x)\cdot\sin 2x = 0 \\ C_1'(x)\cdot(-2)\cdot\sin 2x + C_2'(x)\cdot 2\cdot\cos 2x = \frac{1}{\cos 2x} \end{cases}$$

$$W = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\cdot\sin 2x & 2\cdot\cos 2x \end{vmatrix} = \cos 2x\cdot 2\cdot\cos 2x - [-2\cdot\sin 2x\cdot\sin 2x] =$$

$$= 2\cdot\cos^2 2x + 2\cdot\sin^2 2x = 2\cdot\underbrace{[\cos^2 2x + \sin^2 2x]}_{=1} =$$

$$= 2$$

$$W_{C_1'(x)} = \begin{vmatrix} 0 & \sin 2x \\ \frac{1}{\cos 2x} & 2\cdot\cos 2x \end{vmatrix} = 0 - \frac{\sin 2x}{\cos 2x} = -\frac{\sin 2x}{\cos 2x}$$

$$C_1'(x) = \frac{W_{C_1'(x)}}{W} = -\frac{\sin 2x}{\cos 2x}\cdot\frac{1}{2} = -\frac{\sin 2x}{2\cdot\cos 2x}$$

$$W_{C_2'(x)} = \begin{vmatrix} \cos 2x & 0 \\ -2\cdot\sin 2x & \frac{1}{\cos 2x} \end{vmatrix} = \frac{\cos 2x}{\cos 2x} - 0 = 1$$

$$C_2'(x) = \frac{W_{C_2'(x)}}{W} = \frac{1}{2}$$

$$C_1(x) = \int -\frac{\sin 2x}{2\cdot\cos 2x}\cdot dx = \left\{ \begin{array}{l} \cos 2x = t \\ -2\cdot\sin 2x = dt \\ -\sin 2x = \frac{dt}{2} \end{array} \right\} = \left(\frac{dt}{2\cdot 2\cdot t} = \int \frac{dt}{4\cdot t} = \frac{1}{4}\cdot\int \frac{dt}{t} = \frac{1}{4}\cdot\ln|t| + C_1 = \right.$$

$$C_1(x) = \frac{1}{4}\cdot\ln|\cos 2x| + C_1$$

$$C_2(x) = \int \frac{1}{2}\cdot dx = \frac{1}{2}\cdot\int dx = \frac{1}{2}\cdot x + C_2$$

$$y = C_1(x)\cdot\cos 2x + C_2(x)\cdot\sin 2x$$ ← Constants after variation are inserted to general homogeneous equation

$$y = \left(\frac{1}{4}\cdot\ln|\cos 2x| + C_1\right)\cdot\cos 2x + \left(\frac{1}{2}x + C_2\right)\cdot\sin 2x$$

$$y = \frac{1}{4}\cdot\cos 2x\cdot\ln|\cos 2x| + C_1\cdot\cos 2x + \frac{1}{2}\cdot x\cdot\sin 2x + C_2\cdot\sin 2x$$

$$y = \underbrace{\frac{1}{4}\cdot\cos 2x\cdot\ln|\cos 2x| + \frac{1}{2}\cdot x\cdot\sin 2x}_{\text{Specific inhomogeneous equation}} + \underbrace{C_1\cdot\cos 2x + C_2\cdot\sin 2x}_{\text{General homogeneous equation}}$$

General inhomogeneous equation