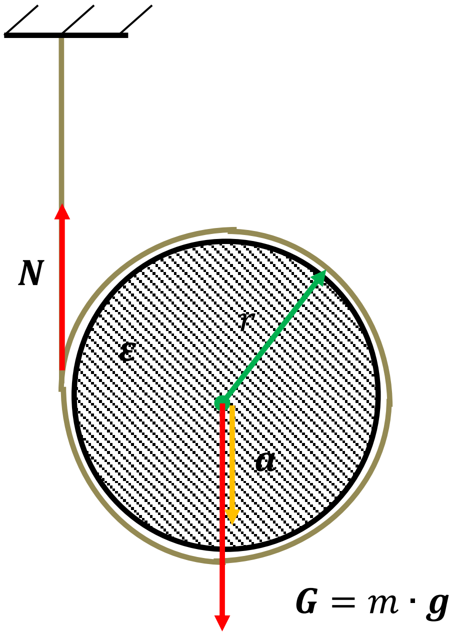
Mechanics dynamics – spool of thread during unrolling

Thread is fixed to the ceiling. Thread has a roller shape. Roller radius is $r[m]$. Mass $m[kg]$ of thread is known. Thread is unrolling to down direction vertically. In example linear acceleration $\boldsymbol{a}[\frac{m}{s^2}]$ has to be calculated. Thread is a roller with twine which was coiled on it. Mass of twine is so small so it is possible to assume it as equal to zero. Thread is a rigid body.



Equilibrium equation for forces in system

$$m \cdot a = G - N$$

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$m \cdot a - force\ (the\ second\ principle\ of\ dynamics)$

$a - linear\ acceleration$

$N - twine\ reaction\ force\ for\ tension$

$G - gravity\ force\ of\ therad$

Equilibrium equation for torques in system

$$I \cdot \varepsilon = N \cdot r$$

$$\frac{1}{2} \cdot m \cdot r^2 \cdot \varepsilon = N \cdot r$$

$I - thread's\ roller\ moment\ of\ inertia$

$\varepsilon - angular\ acceleration$

$r - radius\ of\ roller\ with\ thread$

Angular acceleration is tied with linear acceleration by vector equation

$$\boldsymbol{a} = \boldsymbol{\varepsilon} \times \boldsymbol{r}$$

Value of linear acceleration is given by equation

$$a = \varepsilon \cdot r \cdot \sin \sphericalangle(\boldsymbol{\varepsilon}, \boldsymbol{r})$$

Because angle between vectors $\boldsymbol{\varepsilon}$ and $\boldsymbol{r}$ is $\frac{\pi}{2}$ then value of acceleration is equal

$$a = \varepsilon \cdot r$$

To find value of acceleration a, expression for twine's reaction force is inserted to torque equation.

$$m \cdot a = m \cdot g - N$$

$$N = m \cdot g - m \cdot a$$

$$\frac{1}{2} \cdot m \cdot r^2 \cdot \varepsilon = N \cdot r$$

$$\frac{1}{2} \cdot m \cdot r^2 \cdot \varepsilon = (m \cdot g - m \cdot a) \cdot r$$

$$\frac{1}{2} \cdot m \cdot r \cdot \varepsilon = m \cdot g - m \cdot a$$

$$\frac{1}{2} \cdot m \cdot a = m \cdot g - m \cdot a$$

$$\frac{1}{2} \cdot m \cdot a + m \cdot a = m \cdot g$$

$$1\frac{1}{2} \cdot m \cdot a = m \cdot g$$

$$\frac{3}{2} \cdot m \cdot a = m \cdot g$$

$$\frac{3}{2} \cdot a = g$$

$$a = \frac{2}{3} \cdot g$$