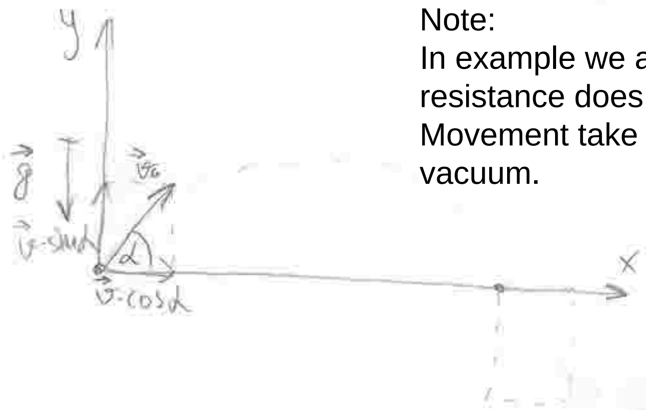
oblique throw of material point

Note:
In example we assume that air resistance does not exist. Movement take place in ideal vacuum.



$$\vec{g} = -\frac{GM}{r^2}\cdot\frac{\vec{r}}{r}$$

$$g = \frac{G\cdot M}{r^2}$$

$$v = v_0 + a\cdot t$$
$$s = v_0\cdot t + \frac{at^2}{2}$$
$$\frac{\sin x}{\cos x} = tg\, x$$

$$\begin{cases} x = v_0\cdot\cos\alpha\cdot t \longrightarrow t = \dfrac{x}{v_0\cdot\cos\alpha} \\ y = v_0\cdot\sin\alpha\, t - \dfrac{gt^2}{2} \end{cases}$$

$$y = v_0\cdot\sin\alpha\cdot\frac{x}{v_0\cdot\cos\alpha} - \frac{g}{2}\cdot\left(\frac{x}{v_0\cos\alpha}\right)^2$$

$$y = \frac{\sin\alpha}{\cos\alpha}\cdot x - \frac{1}{2}\cdot g\cdot\frac{x^2}{v_0^2\cdot\cos^2\alpha}$$

$$y = -\frac{g}{2\cdot v_0^2\cdot\cos^2\alpha}\cdot x^2 + tg\,\alpha\cdot x$$

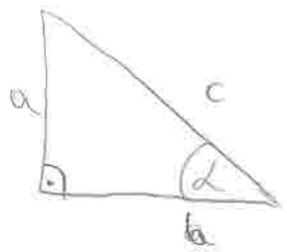
$$y = x\left(tg\,\alpha - \underbrace{\frac{g}{2v_0^2\cdot\cos^2\alpha}}_{c - \text{constant}}\cdot x\right)$$

$$0 = x(tg\,\alpha - c\cdot x)$$

$x_1 = 0$ $\qquad c\cdot x_2 = tg\,\alpha$

$$x_2 = \frac{1}{c}\cdot tg\,\alpha$$

x_2 is the biggest for $\alpha = \frac{\pi}{4}$



$tg\,\alpha = \frac{a}{b}$

if $\alpha = \frac{\pi}{4}$ then $a = b$

$\Rightarrow tg\,\alpha = 1$

if $b \to 0$ then $tg\,\alpha \to \infty$

$$f(x) = a\cdot x^2 + bx + c$$
$$\Delta = b^2 - 4a\cdot c$$
$$x_1 = \frac{-b - \sqrt{\Delta}}{2a}$$
$$x_2 = \frac{-b + \sqrt{\Delta}}{2a}$$

$$\Delta = tg^2\,\alpha$$

$$x_2 = (-(tg\,\alpha) - tg\,\alpha)\cdot\frac{1}{2}\cdot\frac{-2\cdot v_0^2\cdot\cos^2\alpha}{g}$$

$$x = 2\frac{\sin\alpha}{\cos\alpha}\cdot\frac{v_0^2\cdot\cos^2\alpha}{g}$$

$$y = -\frac{g \cdot x^2}{2 \cdot v_0^2 \cdot \cos^2\alpha} + tg\,\alpha \cdot x$$

$$0 = -\frac{g \cdot x^2}{2 \cdot v_0^2 \cdot \cos^2\alpha} + tg\,\alpha \cdot x$$

$$\Delta = b^2 - 4ac$$

$$\Delta = tg^2\alpha$$

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-2\,tg\,\alpha}{2a} = -\cancel{2} \cdot \frac{\sin\alpha}{\cancel{\cos\alpha}} \cdot \frac{-2 \cdot v_0^2 \cdot \cos^{\cancel{2}}\alpha}{g} \cdot \frac{1}{\cancel{2}}$$

$$x_1 = \sin\alpha \cdot \frac{2 \cdot v_0^2 \cdot \cos\alpha}{g}$$

$$\left\{ 2 \cdot \sin\alpha \cdot \cos\alpha = \sin 2\alpha \right.$$

$$x_1 = \frac{v_0^2}{g} \cdot 2 \cdot \sin\alpha \cdot \cos\alpha$$

$$x_1 = \frac{v_0^2}{g} \cdot \sin 2\alpha$$

$x_1 = x_{max}$ for $\alpha = \frac{\pi}{4} \rightarrow \sin 2 \cdot \alpha = 1$

$x_2 = 0$