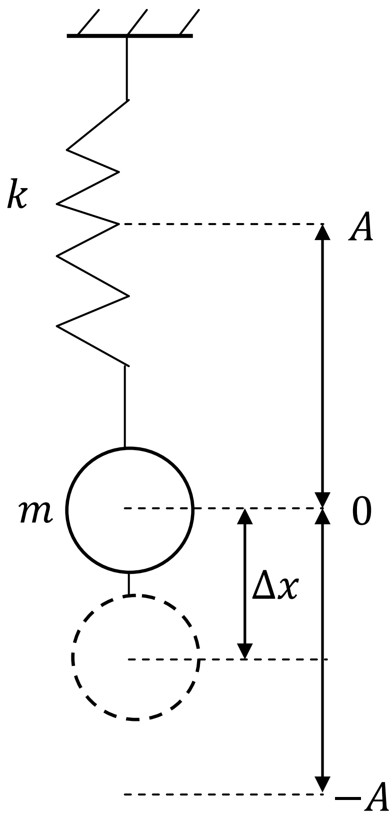
Movement equation of harmonic oscillator



Material point with mass m[kg] is hanged via spring to the ceiling. Spring has stiffness $k\left[\frac{N}{m}\right]$. We assume that there is no gravity field and also that motion takes place in vacuum. At time t=0 material point is equilibrium position. After a while material point is precipitated from equilibrium position and on material point works force which is proportional to spring's stiffness and deflection from equilibrium position.

We write motion equation

$$m \cdot a = -k \cdot \Delta x$$

Force which works on material point has negative sign because its direction is opposite to direction of deflection.

$$m \cdot \frac{d^2x}{dt^2} = -k \cdot \Delta x$$

$$m \cdot \frac{d^2x}{dt^2} + k \cdot \Delta x = 0$$

$$\frac{d^2x}{dt^2} + \frac{k}{m} \cdot \Delta x = 0$$

$$\frac{d^2x}{dt^2} + \omega^2 \cdot \Delta x = 0; where\ \omega^2 = \frac{k}{m}$$

As you see above motion equation of harmonic oscillator is differential equation. Characteristic equation for differential equation is following

$$r^2 + \omega^2 = 0$$

Quadratic equation above has two solutions

$$r_{0\_1} = i \cdot \omega$$

$$r_{0\_2} = -i \cdot \omega$$

$$i^2 = -1;\ where\ i\ is\ imaginary\ unit$$

As you know from mathematics general solution of differential equation from our example is in form

$$x(t) = C_1 \cdot e^{i\cdot\omega\cdot t} + C_2 \cdot e^{-i\cdot\omega\cdot t}$$

Now we have to calculate value of constants C_1 and C_2. We will use border conditions. First condition is $x(t = 0) = 0$.

$$0 = C_1 \cdot e^{i\cdot\omega\cdot 0} + C_2 \cdot e^{-i\cdot\omega\cdot 0} \rightarrow C_2 = -C_1$$

$$x(t) = C_1 \cdot e^{i\cdot\omega\cdot t} - C_1 \cdot e^{-i\cdot\omega\cdot t}$$

We will change representation from exponential form to trigonometric form.

$$x(t) = C_1 \cdot (\cos\omega \cdot t + i \cdot \sin\omega \cdot t) - C_1 \cdot (\cos\omega \cdot t - i \cdot \sin\omega \cdot t)$$

$$x(t) = C_1 \cdot \cos\omega \cdot t + i \cdot C_1 \cdot \sin\omega \cdot t - C_1 \cdot \cos\omega \cdot t + i \cdot C_1 \cdot \sin\omega \cdot t$$

$$x(t) = i \cdot C_1 \cdot \sin\omega \cdot t + i \cdot C_1 \cdot \sin\omega \cdot t$$

$$x(t) = 2 \cdot i \cdot C_1 \cdot \sin\omega \cdot t$$

$$x(t) = A \cdot \sin\omega \cdot t$$

Where

$A - amplitude$
$\omega = 2 \cdot \pi \cdot f - pulsation$

Velocity is derivative of road

$$\frac{dx}{dt} = \dot{x}(t) = A \cdot \ \omega \cdot \cos\omega \cdot t$$

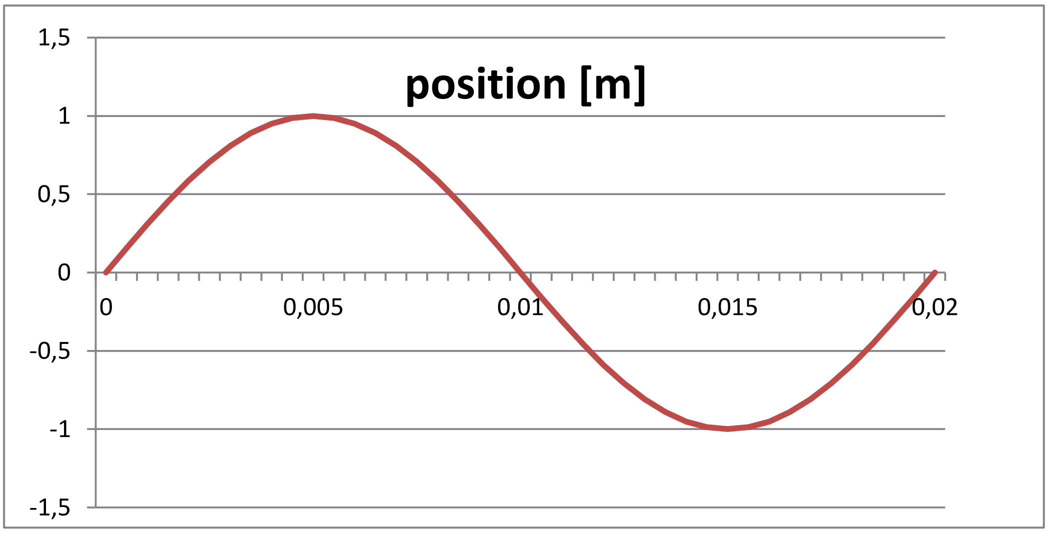
Acceleration is derivative of velocity

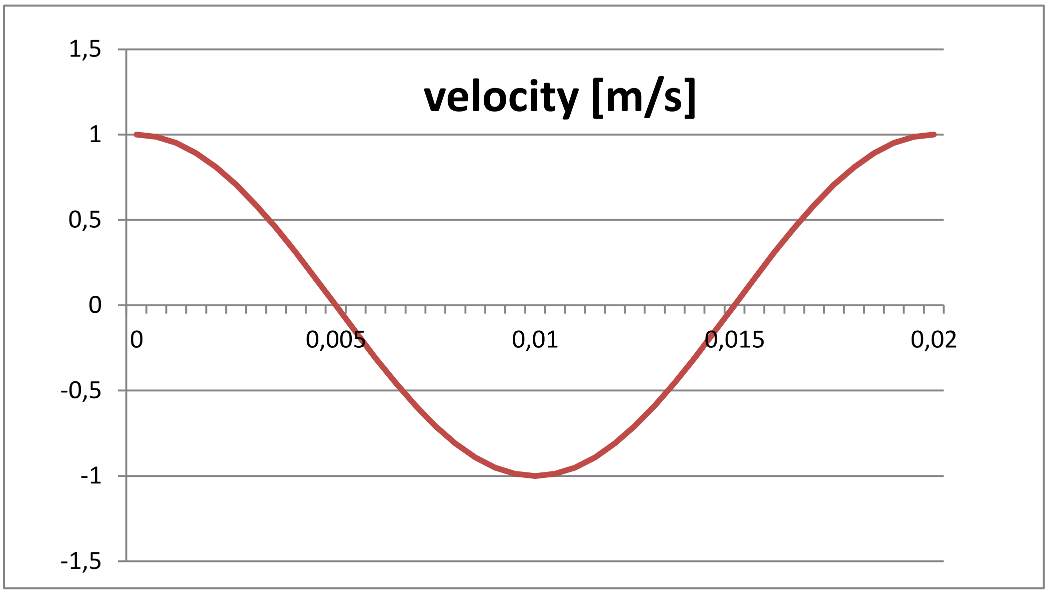
$$\frac{d^2x}{dt^2} = \ddot{x}(t) = -A \cdot \ \omega^2 \cdot \sin\omega \cdot t$$

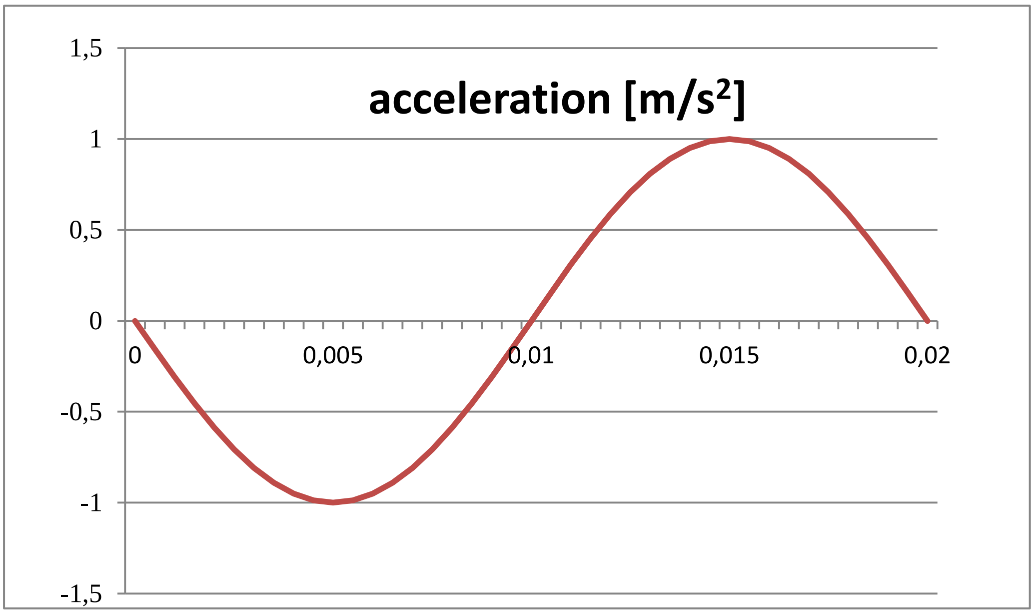
On next page are placed example plots of position $x(t)$, velocity $\dot{x}(t)$, and acceleration $\ddot{x}(t)$. Plots were draw for input data:

$$A = 1[m]; \ \omega = 100 \cdot \Pi \left[\frac{rad}{s}\right]; T = 0{,}02[s]$$

http://www.mbstudent.com/physics-examples.html







Drawing 1. Plots of position, velocity and acceleration. All are plotted as a function of time.