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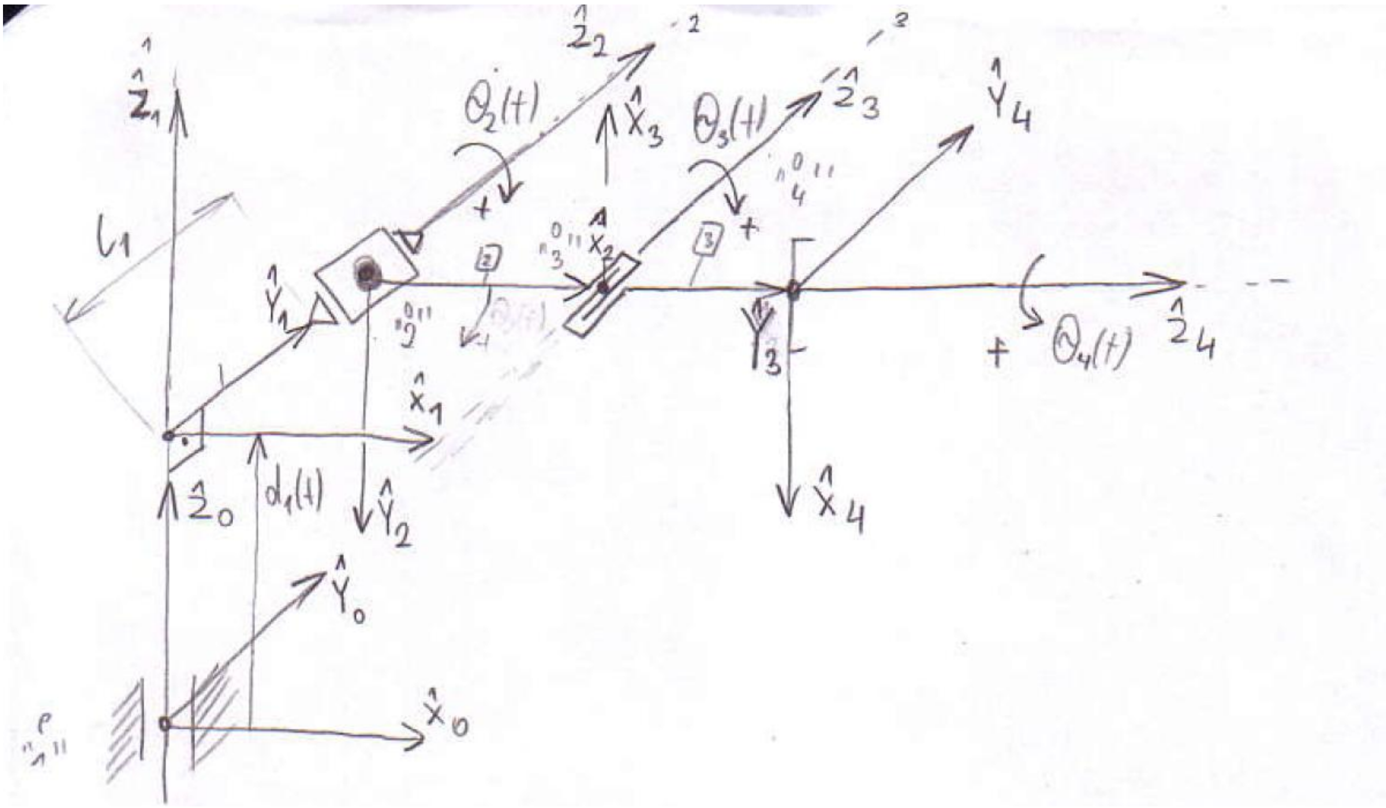
Robotics – forward kinematics



Drawing 1. Model of manipulator.

Table with Denavit-Hartenberg's parameters

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	$d_1(t)$	0
2	$-\frac{\pi}{2}$	0	l_1	$\theta_2(t)$
3	0	l_2	0	$\theta_3(t)$
4	$-\frac{\pi}{2}$	0	l_3	$\theta_4(t)$

$$ {}^{0}_{1}\boldsymbol{T} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & a_0 \\ s\theta_1 \cdot c\alpha_0 & c\theta_1 \cdot c\alpha_0 & -s\alpha_0 & -s\alpha_0 \cdot d_1 \\ s\theta_1 \cdot s\alpha_0 & c\theta_1 \cdot s\alpha_0 & c\alpha_0 & c\alpha_0 \cdot d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{1}\boldsymbol{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1(t) \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

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$$ {}_{2}^{1}\boldsymbol{T} = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_1 \\ s\theta_2 \cdot c\alpha_1 & c\theta_2 \cdot c\alpha_1 & -s\alpha_1 & -s\alpha_1 \cdot d_2 \\ s\theta_2 \cdot s\alpha_1 & c\theta_2 \cdot s\alpha_1 & c\alpha_1 & c\alpha_1 \cdot d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_{2}^{1}\boldsymbol{T} = \begin{bmatrix} c\theta_2(t) & -s\theta_2(t) & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ -s\theta_2(t) & -c\theta_2(t) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_{3}^{2}\boldsymbol{T} = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 \cdot c\alpha_2 & c\theta_3 \cdot c\alpha_2 & -s\alpha_2 & -s\alpha_2 \cdot d_3 \\ s\theta_3 \cdot s\alpha_2 & c\theta_3 \cdot s\alpha_2 & c\alpha_2 & c\alpha_2 \cdot d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_{3}^{2}\boldsymbol{T} = \begin{bmatrix} c\theta_3(t) & -s\theta_3(t) & 0 & l_2 \\ s\theta_3(t) & c\theta_3(t) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_{4}^{3}\boldsymbol{T} = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ s\theta_4 \cdot c\alpha_3 & c\theta_4 \cdot c\alpha_3 & -s\alpha_3 & -s\alpha_3 \cdot d_4 \\ s\theta_4 \cdot s\alpha_3 & c\theta_4 \cdot s\alpha_3 & c\alpha_3 & c\alpha_3 \cdot d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_{4}^{3}\boldsymbol{T} = \begin{bmatrix} c\theta_4(t) & -s\theta_4(t) & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ -s\theta_4(t) & -c\theta_4(t) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \boldsymbol{T}_4^0 = \boldsymbol{T}_1^0 \cdot \boldsymbol{T}_2^1 \cdot \boldsymbol{T}_3^2 \cdot \boldsymbol{T}_4^3 $$

Position of any vector $\boldsymbol{P}$ in coordinate system "$i = 4$" defined in coordinate system "$i = 0$", is given by equation.

$$ \boldsymbol{P}^0 = \boldsymbol{T}_4^0 \cdot \boldsymbol{P}^4 $$