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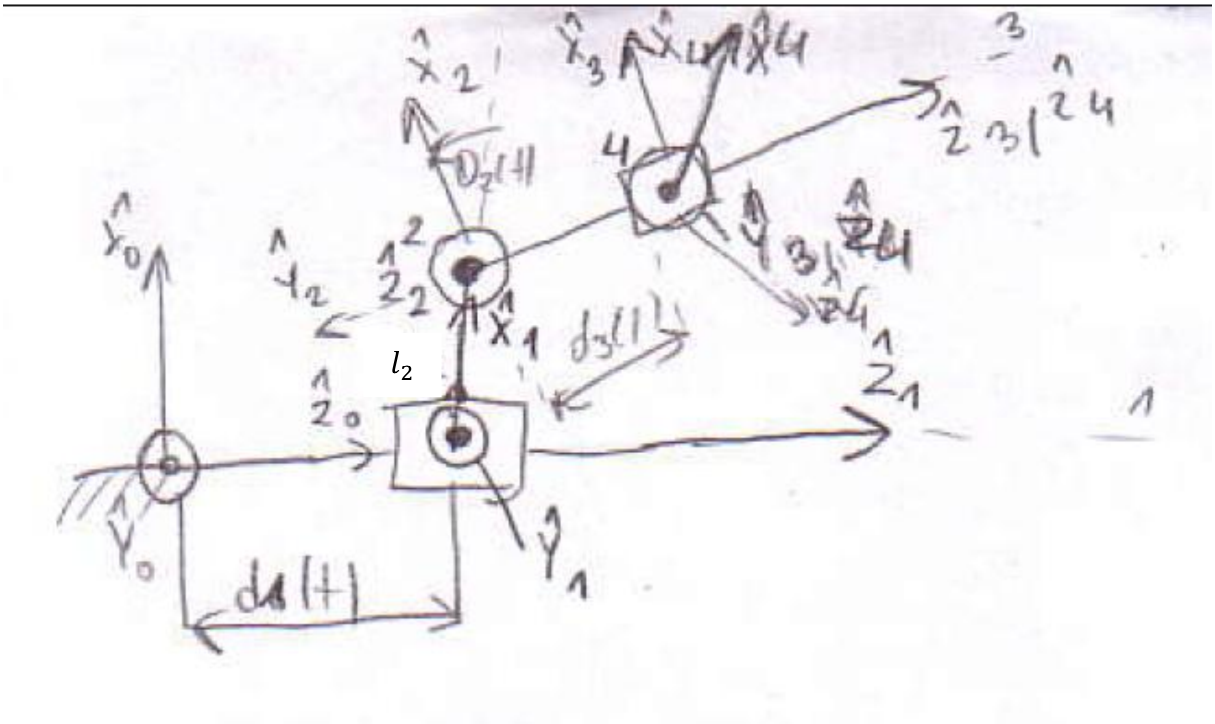
Robotics – forward kinematics



Drawing 1. Model of manipulator.

Table with Denavit-Hartenberg's parameters

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	$d_1(t)$	0
2	$-\frac{\pi}{2}$	l_2	0	$\theta_2(t)$
3	$\frac{\pi}{2}$	0	$d_3(t)$	0
4	$-\frac{\pi}{2}$	0	0	$\theta_4(t)$

Matrix ${}^{i-1}_{\ \ i}T$ describes transformation form coordinate system „i" to coordinate system „$i-1$"

$$
{}^{i-1}_{\ \ i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i \cdot c\alpha_{i-1} & c\theta_i \cdot c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} \cdot d_i \\ s\theta_i \cdot s\alpha_{i-1} & c\theta_i \cdot s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} \cdot d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

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In transformation matrix we use shortened notation of trigonometric functions

$$s\theta_i \rightarrow sin\theta_i \; oraz \; c\theta_i \rightarrow cos\theta_i$$

$$s\theta_i \rightarrow sin\theta_i \; and \;\; c\theta_i \rightarrow cos\theta_i$$

Transformation matrix which describes transformation from system "$i = 1$" to "$i = 0$".

$$ {}^{0}_{1}\boldsymbol{T} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & a_0 \\ s\theta_1 \cdot c\alpha_0 & c\theta_1 \cdot c\alpha_0 & -s\alpha_0 & -s\alpha_0 \cdot d_1 \\ s\theta_1 \cdot s\alpha_0 & c\theta_1 \cdot s\alpha_0 & c\alpha_0 & c\alpha_0 \cdot d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{1}\boldsymbol{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1(t) \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Transformation matrix which describes transformation from system "$i = 2$" to "$i = 1$".

$$ {}^{1}_{2}\boldsymbol{T} = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_1 \\ s\theta_2 \cdot c\alpha_1 & c\theta_2 \cdot c\alpha_1 & -s\alpha_1 & -s\alpha_1 \cdot d_2 \\ s\theta_2 \cdot s\alpha_1 & c\theta_2 \cdot s\alpha_1 & c\alpha_1 & c\alpha_1 \cdot d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}\boldsymbol{T} = \begin{bmatrix} c\theta_2(t) & -s\theta_2(t) & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ -s\theta_2(t) & -c\theta_2(t) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Transformation matrix which describes transformation from system "$i = 3$" to "$i = 2$".

$$ {}^{2}_{3}\boldsymbol{T} = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 \cdot c\alpha_2 & c\theta_3 \cdot c\alpha_2 & -s\alpha_2 & -s\alpha_2 \cdot d_3 \\ s\theta_3 \cdot s\alpha_2 & c\theta_3 \cdot s\alpha_2 & c\alpha_2 & c\alpha_2 \cdot d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}\boldsymbol{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_3(t) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Transformation matrix which describes transformation from system "$i = 4$" to "$i = 3$".

$$ {}^{3}_{4}\boldsymbol{T} = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ s\theta_4 \cdot c\alpha_3 & c\theta_4 \cdot c\alpha_3 & -s\alpha_3 & -s\alpha_3 \cdot d_4 \\ s\theta_4 \cdot s\alpha_3 & c\theta_4 \cdot s\alpha_3 & c\alpha_3 & c\alpha_3 \cdot d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_4^3\boldsymbol{T} = \begin{bmatrix} c\theta_4(t) & -s\theta_4(t) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_4(t) & -c\theta_4(t) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \boldsymbol{T}_4^0 = \boldsymbol{T}_1^0 \cdot \boldsymbol{T}_2^1 \cdot \boldsymbol{T}_3^2 \cdot \boldsymbol{T}_4^3 $$

Position of any vector $\boldsymbol{P}$ in coordinate system "$i = 4$" defined in coordinate system "$i = 0$", is given by equation.

$$ \boldsymbol{P}^0 = \boldsymbol{T}_4^0 \cdot \boldsymbol{P}^4 $$