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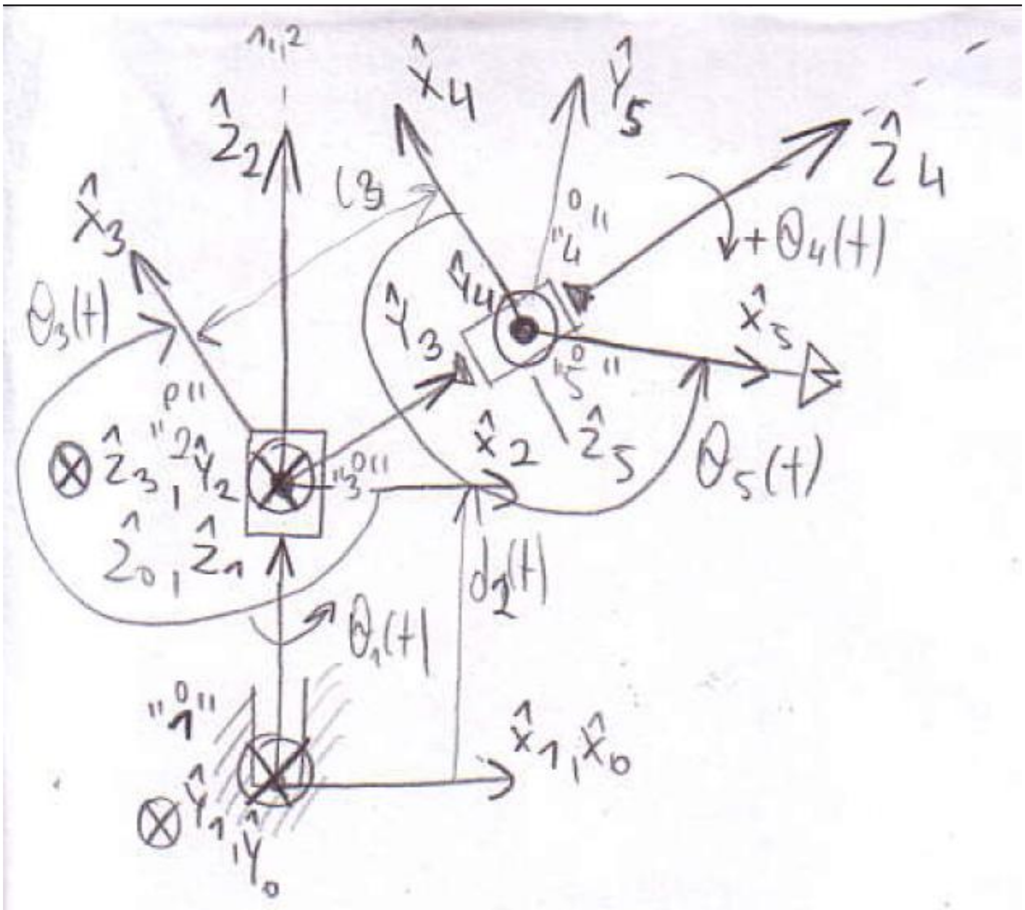
Robotics – forward kinematics



Drawing 1. Model of manipulator.

Table with Denavit-Hartenberg's parameters

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	$\theta_1(t)$
2	0	0	$d_2(t)$	0
3	$-\frac{\pi}{2}$	0	0	$\theta_3(t)$
4	$-\frac{\pi}{2}$	0	l_3	$\theta_4(t)$
5	$-\frac{\pi}{2}$	0	0	$\theta_5(t)$

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Matrix ${}^{i-1}_{i}T$ describes transformation form coordinate system „i" to coordinate system „$i-1$"

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i \cdot c\alpha_{i-1} & c\theta_i \cdot c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} \cdot d_i \\ s\theta_i \cdot s\alpha_{i-1} & c\theta_i \cdot s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} \cdot d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

In transformation matrix we use shortened notation of trigonometric functions

$$ s\theta_i \rightarrow sin\theta_i \ oraz \ c\theta_i \rightarrow cos\theta_i $$

$$ s\theta_i \rightarrow sin\theta_i \ and \ \ c\theta_i \rightarrow cos\theta_i $$

Transformation matrix which describes transformation from system "$i = 1$" to "$i = 0$".

$$ {}^{0}_{1}\boldsymbol{T} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & a_0 \\ s\theta_1 \cdot c\alpha_0 & c\theta_1 \cdot c\alpha_0 & -s\alpha_0 & -s\alpha_0 \cdot d_1 \\ s\theta_1 \cdot s\alpha_0 & c\theta_1 \cdot s\alpha_0 & c\alpha_0 & c\alpha_0 \cdot d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{1}\boldsymbol{T} = \begin{bmatrix} c\theta_1(t) & -s\theta_1(t) & 0 & 0 \\ s\theta_1(t) & c\theta_1(t) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Transformation matrix which describes transformation from system "$i = 2$" to "$i = 1$".

$$ {}^{1}_{2}\boldsymbol{T} = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_1 \\ s\theta_2 \cdot c\alpha_1 & c\theta_2 \cdot c\alpha_1 & -s\alpha_1 & -s\alpha_1 \cdot d_2 \\ s\theta_2 \cdot s\alpha_1 & c\theta_2 \cdot s\alpha_1 & c\alpha_1 & c\alpha_1 \cdot d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}\boldsymbol{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2(t) \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Transformation matrix which describes transformation from system "$i = 3$" to "$i = 2$".

$$ {}^{2}_{3}\boldsymbol{T} = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 \cdot c\alpha_2 & c\theta_3 \cdot c\alpha_2 & -s\alpha_2 & -s\alpha_2 \cdot d_3 \\ s\theta_3 \cdot s\alpha_2 & c\theta_3 \cdot s\alpha_2 & c\alpha_2 & c\alpha_2 \cdot d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}\boldsymbol{T} = \begin{bmatrix} c\theta_3(t) & -s\theta_3(t) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_3(t) & -c\theta_3(t) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

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Transformation matrix which describes transformation from system "$i = 4$" to "$i = 3$".

$$
{}^{3}_{4}\boldsymbol{T} = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ s\theta_4 \cdot c\alpha_3 & c\theta_4 \cdot c\alpha_3 & -s\alpha_3 & -s\alpha_3 \cdot d_4 \\ s\theta_4 \cdot s\alpha_3 & c\theta_4 \cdot s\alpha_3 & c\alpha_3 & c\alpha_3 \cdot d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{3}_{4}\boldsymbol{T} = \begin{bmatrix} c\theta_4(t) & -s\theta_4(t) & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ -s\theta_4(t) & -c\theta_4(t) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Transformation matrix which describes transformation from system "$i = 5$" to "$i = 4$".

$$
{}^{4}_{5}\boldsymbol{T} = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & a_4 \\ s\theta_5 \cdot c\alpha_4 & c\theta_5 \cdot c\alpha_4 & -s\alpha_4 & -s\alpha_4 \cdot d_5 \\ s\theta_5 \cdot s\alpha_4 & c\theta_5 \cdot s\alpha_4 & c\alpha_4 & c\alpha_4 \cdot d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{4}_{5}\boldsymbol{T} = \begin{bmatrix} c\theta_5(t) & -s\theta_5(t) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_5(t) & -c\theta_5(t) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\boldsymbol{T}_5^0 = \boldsymbol{T}_1^0 \cdot \boldsymbol{T}_2^1 \cdot \boldsymbol{T}_3^2 \cdot \boldsymbol{T}_4^3 \cdot \boldsymbol{T}_5^4
$$

Position of any vector $\boldsymbol{P}$ in coordinate system "$i = 5$" defined in coordinate system "$i = 0$", is given by equation.

$$
\boldsymbol{P}^0 = \boldsymbol{T}_5^0 \cdot \boldsymbol{P}^5
$$